MPM1D: Principles of Mathematics

Rearranging Formulae

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Solving Equations Involving Fractions

Recap

Solve $\frac{5}{6}(x-4)=\frac{3}{10}(2x-1)$ algebraically.

The LCM of 6 and 10 is 30, so multiply both sides by 30, distribute, and solve.

$$\begin{aligned}30\cdot\tfrac{5}{6}(x-4)&=30\cdot\tfrac{3}{10}(2x-1)\\25(x-4)&=9(2x-1)\\25x-100&=18x-9\\25x-18x&=100-9\\7x&=91\\x&=13\end{aligned}$$

Rearranging Formulae

Consider the formula for the area of a rectangle, $A=\ell\cdot w$.

If we want to determine the area for a given length and width, we simply substitute those values into ℓ and w and multiply them together.

If we want to know the value of one of the dimensions, however, we need to rearrange the formula, isolating the desired variable.

For instance, if we want to calculate a rectangle's length, given its width and area, we can *isolate* ℓ by dividing both sides by w.

$$\begin{aligned}\frac{A}{w}&=\frac{\ell\cdot \cancel{w}}{\cancel{w}}\\\frac{A}{w}&=\ell\end{aligned}$$

Rearranging Formulae

This new formula allows us to calculate the length directly.

There are many situations in which we need to rearrange formulae, depending on which value is desired (and which values are given).

We can do this using the same algebraic techniques that we used for solving equations.

Rearranging Formulae

Example

Ohm's Law relates the voltage, V (volts), in a conductor to both the current, I (amps), and the resistance, R (ohms), according to the relationship $V=IR$. If the voltage is 12 V, and the resistance is 1.5 Ω, what is the current?

Divide both sides of the equation by R to isolate I.

$$\begin{aligned}\frac{V}{R}&=\frac{I\cancel{R}}{\cancel{R}}\\\frac{V}{R}&=I\end{aligned}$$

Substitute the values 12 and 1.5 into the equation.

$$\begin{aligned}I&=\frac{12}{1.5}\\I&=8\text{ A}\end{aligned}$$

Rearranging Formulae

Example

The formula to convert a temperature in Celsius, C (degrees), to Fahrenheit, F (degrees), is $F=\frac{9}{5}C+32$. Use this formula to convert a temperature of 77°F to Celsius.

First, isolate C.

$$\begin{aligned}F-32&=\tfrac{9}{5}C\\5(F-32)&=9C\\\tfrac{5}{9}(F-32)&=C\end{aligned}$$

Now, substitute 77 into F.

$$\begin{aligned}C&=\tfrac{5}{9}(77-32)\\C&=\tfrac{5}{9}(45)\\C&=25^\circ\text{C}\end{aligned}$$

Rearranging Formulae

Example

The volume of a cylinder is given by $V = \pi r^2 h$, where r is the radius and h is the height. Determine the radius of a cylinder with a height of 14 cm and a volume of 320 cm^3.

Begin by isolating r^2, then take the square root to isolate r.

$$\frac{V}{\pi h} = r^2$$
$$\sqrt{\frac{V}{\pi h}} = r$$

Now substitute 14 and 320 into h and V respectively.

$$r = \sqrt{\frac{320}{14\pi}}$$
$$r \approx 2.7 \text{ cm}$$

Rearranging Formulae

Example

A simple pendulum of given length, ℓ metres, has a *period*, T seconds, given by the equation $T = 2\pi\sqrt{\frac{\ell}{g}}$, where g is the acceleration due to gravity (typically 9.8 m/s^2). What is the length of a pendulum that has a period of 3 seconds?

To isolate ℓ, start by dividing both sides by 2π.

$$\frac{T}{2\pi} = \frac{\cancel{2\pi}\sqrt{\frac{\ell}{g}}}{\cancel{2\pi}}$$

Rearranging Formulae

Square both sides to cancel out the square root.

$$\left(\frac{T}{2\pi}\right)^2 = \frac{\ell}{g}$$

Multiply both sides by g to isolate ℓ.

$$g \cdot \left(\frac{T}{2\pi}\right)^2 = \frac{\cancel{g} \cdot \ell}{\cancel{g}}$$
$$g \cdot \left(\frac{T}{2\pi}\right)^2 = \ell$$

Now substitute $g = 9.8$ and $T = 3$ into the formula.

$$\ell = 9.8 \cdot \left(\frac{3}{2\pi}\right)^2$$
$$\ell \approx 2.23 \text{ m}$$

Questions?