MPM1D: Principles of Mathematics

Number Systems

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Number Systems

Most of the time in mathematics, we are concerned with a number's *value*.

Another important aspect of a number is its *type*.

For example, it may make sense to talk about eating 2.5 pizzas, but not so much to talk about 2.5 people.

In these cases, we can categorize numbers into various *number systems*.

Number Systems

The *natural numbers* are those used for counting: 1, 2, 3, and so forth.

The mathematical symbol for the set of natural numbers is the letter N.

Whole numbers are generally regarded as the set of natural numbers, along with zero. The symbol W may be used, but is not too common.

Some sources use the terms "natural numbers" and "whole numbers" interchangeably, but in this course we will distinguish between the two.

Number Systems

The *integers* are comprised of both positive and negative whole numbers.

Examples of integers are 5, -17, 0, 13 512 742, etc.

The symbol for the integers is Z, from the German *zahlen* for "numbers." Some texts use I, but this is non-standard and should be avoided.

Integers are often broken down into smaller groups:

- positive integers: 1, 2, 3, ... (aka natural numbers)
- negative integers: -1, -2, -3, ...
- non-negative integers: 0, 1, 2, ... (aka whole numbers)
- non-positive integers: 0, -1, -2, ...

Number Systems

A number that can be written as a ratio of two integers (what we would generally call a *common fraction*) is a *rational number*.

For example, $\frac{1}{3}$, $\frac{7}{2}$ and $-\frac{4}{19}$ are all rational numbers.

The number 5 is a rational number, since $5 = \frac{5}{1}$.

The number 0.5 is a rational number, since $0.5 = \frac{5}{10} = \frac{1}{2}$.

The symbol for the rational numbers is Q, from *quotient*.

Numbers that are not rational are *irrational*.

The numbers $\sqrt{2}$, π and 0.123456789101112131415... are all irrational numbers.

There is no standard symbol for irrational numbers, but $\overline{Q}$ is relatively common.

Number Systems

The *real numbers* include all of the previous number systems: naturals, wholes, integers, rationals and irrationals.

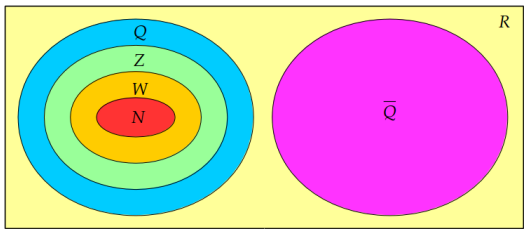
Formally, we say that the previous number systems are *subsets* of the real number system.

There are other number systems "beyond" the real numbers. For example, *complex numbers* (symbol C) are made of two components – a real component, and an *imaginary* component.

These number systems may be covered in greater detail in later courses.

Number Systems

A Venn diagram showing the relationship between the previous number systems is below.



Note that the rational numbers and the irrational numbers do not overlap. We refer to these sets of numbers as *disjoint*.

Number Systems

Example

To which number system(s) does the number 1.2 belong?

Since 1.2 can be represented as $\frac{12}{10} = \frac{6}{5}$, 1.2 is a rational number.

It is also a real number, since all rationals are reals.

Example

To which number system(s) does the number -7 belong?

-7 is an integer, a rational number (since $-7 = -\frac{7}{1}$) and a real number.

Number Systems

Example

To which number system(s) does the number $0.\overline{17}$ belong?

The horizontal bar indicates that the 17 is repeated indefinitely. That is, $0.\overline{17} = 0.171717\ldots = \frac{17}{99}$.

Thus, $0.\overline{17}$ is rational and real.

Example

To which number system(s) does the number $\frac{1+\sqrt{5}}{2}$ belong?

This value, approximately 1.618, is often referred to as the "golden ratio", and given the symbol ϕ (phi).

Since its decimal expansion does not repeat, it is irrational and real.

Questions?