MPM1D: Principles of Mathematics

Solving Linear Systems Graphically

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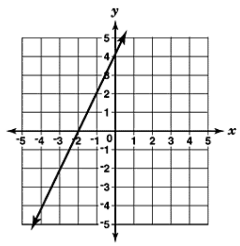


Equations of Lines

Recap

Determine whether the lines $y = \frac{5}{3}x + 4$ and $5x - 3y = 24$ are coincident, parallel and distinct, or neither.

Rearrange the second equation into slope-intercept form to compare the slopes.

$$\begin{aligned} 5x - 3y &= 24 \\ -3y &= -5x + 24 \\ y &= \tfrac{5}{3}x - 8 \end{aligned}$$

Since the slopes are the same, the lines are either coincident or parallel and distinct.

Since the y-intercepts are different, the two lines must be parallel and distinct.

Linear Systems

Two or more linear equations that are considered simultaneously is called a *linear system*.

These may be expressed in either slope-intercept ($y = mx + b$) or standard ($Ax + By = C$) form.

The solution to a linear system is a set of values that satisfies *all* equations.

For example, the linear system with equations $y = 5x - 12$ and $y = 3x - 4$ has the solution $x = 4$, $y = 8$, since these values make both equations true.

$$8 = 5(4) - 12$$
$$8 = 3(4) - 4$$

Linear Systems

When graphed, the linear relations $y = 5x - 12$ and $y = 3x - 4$ make two straight lines.

Since these lines have different slopes, they must intersect at a unique point.

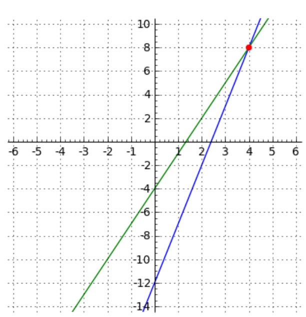
Since $x = 4$, $y = 8$ is a solution to the linear system, this corresponds to the point $(4, 8)$ on the graph.

Solving Linear Systems Graphically

When represented graphically, the solution to a linear system is the *point of intersection* (POI) of the lines.

Linear Systems

A graph of the two relations is below, showing the point of intersection at $(4, 8)$.



Linear Systems

Sometimes, two lines are parallel to each other.

Since two parallel lines will never intersect, there will not be a point of intersection.

This implies that a linear system represented by two parallel lines will have no solution.

In other cases, the same line may be described in multiple ways (i.e. in slope-intercept form and in standard form).

If both lines are coincident, there will be an infinite number of solutions, since any values of x and y that satisfy the first equation will also satisfy the second.

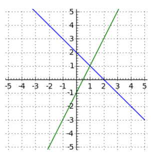
Linear Systems

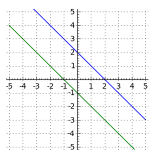
Number of Solutions to a Linear System

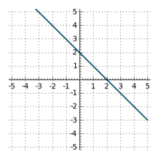
For a linear system, the number of solutions will be:

- one, if the lines are not parallel,
- zero, if the lines are parallel and distinct, or
- infinite, if the lines are coincident.

Each scenario is depicted below.







Linear Systems

Example

Solve the linear system with equations $y = 2x - 5$ and $y = -\frac{1}{3}x + 2$.

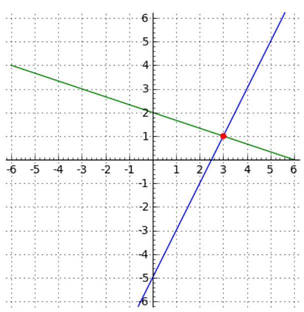
Since the two lines have different slopes, they *must* intersect at a single point.

The first line has a y-intercept at -5. Beginning at this point, move up 2 units and right 1 unit.

The second line has a y-intercept at 2. Beginning at this point, move down 1 unit and right 3 units.

Linear Systems

A graph of the lines is below, showing a point of intersection at $(3, 1)$. The solution is $x = 3$, $y = 1$.



Linear Systems

Example

Solve the linear system with equations $y = 3x + 5$ and $y = 2x + 3$.

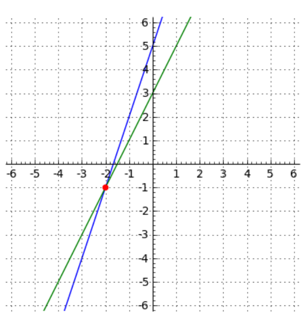
Even though both slopes are positive, they are different, so the lines will intersect at a single point.

The first line has a y-intercept at 5. Beginning at this point, move up 3 units and right 1 unit.

The second line has a y-intercept at 3. Beginning at this point, move up 2 units and right 1 unit.

Linear Systems

A graph of the lines is below, showing a point of intersection at $(-2, -1)$. The solution is $x = -2$, $y = -1$.



Linear Systems

Example

Solve the linear system with equations $y = \frac{3}{2}x - 4$ and $3x - 2y = -2$.

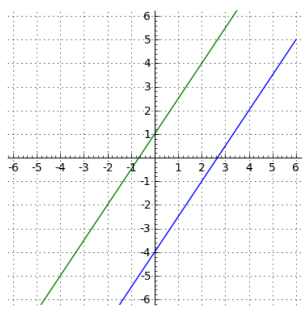
Since the second line is expressed using standard form, and since 3 is not a factor of -2, convert it to slope-intercept form.

$$\begin{aligned} 3x - 2y &= -2 \\ -2y &= -3x - 2 \\ y &= \tfrac{3}{2}x + 1 \end{aligned}$$

Since the slopes of the lines are identical, but the y-intercepts are different, the lines must be parallel and distinct. Therefore, there is no solution.

Linear Systems

A graph of the lines is below, confirming that the two lines are parallel.



Linear Systems

Example

Solve the linear system with equations $y = -\frac{1}{2}x + 2$ and $x + 2y = 4$.

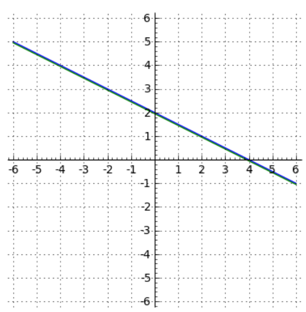
Again, convert the second line to slope-intercept form.

$$\begin{aligned} x + 2y &= 4 \\ 2y &= -x + 4 \\ y &= -\tfrac{1}{2}x + 2 \end{aligned}$$

Since the slopes of the lines are identical, and the y-intercepts are the same, the lines must be coincident. Therefore, there are an infinite number of solutions.

Linear Systems

A graph of the lines is below. Note that the intercepts for $x + 2y = 4$ are $(4, 0)$ and $(0, 2)$.



Questions?