MPM1D: Principles of Mathematics

Graphing Lines Using Intercepts

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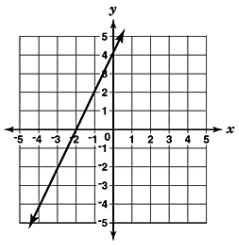


Standard Form of a Line

Recap

Graph the line $5x + 3y = 6$.

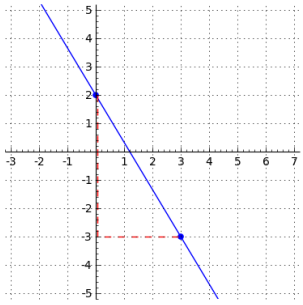
Isolate y to rewrite the equation in slope-intercept form.

$$\begin{aligned} 5x + 3y &= 6 \\ 3y &= -5x + 6 \\ \frac{3y}{3} &= \frac{-5x}{3} + \frac{6}{3} \\ y &= -\tfrac{5}{3}x + 2 \end{aligned}$$

The y-intercept is at $(0, 2)$ and the slope of the line is $-\frac{5}{3}$.

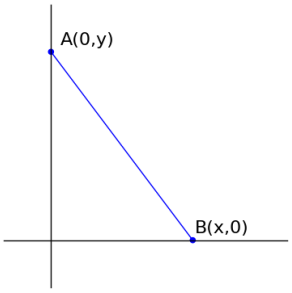
Standard Form of a Line

A graph of $5x + 3y = 6$, or $y = -\frac{5}{3}x + 2$, is below.



Graphing Lines Using Intercepts

Recall that any y-intercept has an x-coordinate of zero, and that any x-intercept has a y-coordinate of zero.



Graphing Lines Using Intercepts

To determine the y-intercept of a linear relation, set $x = 0$ and solve for y.

Similarly, to find the x-intercept of a linear relation, set $y = 0$ and solve for x.

These two points can be plotted on the Cartesian Plane to graph the line.

If a linear relation is expressed in slope-intercept form, then the y-intercept is already given, and only the x-intercept needs to be determined.

A linear relation expressed in standard form, however, will require both.

Graphing Lines Using Intercepts

Example

Determine the intercepts of $y = -2x + 8$, and graph the line.

The y-intercept is given, and is at $(0, 8)$.

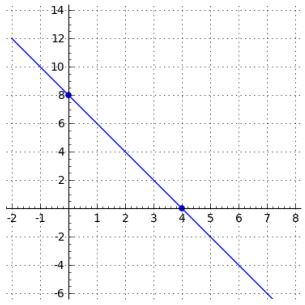
To find the x-intercept, set $y = 0$ and solve for x.

$$\begin{aligned} 0 &= -2x + 8 \\ 2x &= 8 \\ x &= 4 \end{aligned}$$

Therefore, the x-intercept is at $(4, 0)$.

Graphing Lines Using Intercepts

A graph of $y = -2x + 8$ is below.



Graphing Lines Using Intercepts

Example

Determine the intercepts of $5x - 4y = 20$, and graph the line.

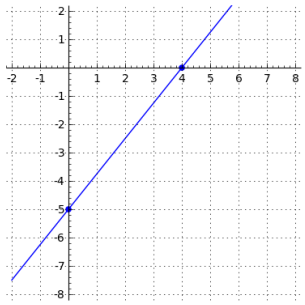
Set $x = 0$ to find the y-intercept, and set $y = 0$ to find the x-intercept.

$$\begin{aligned} 5(0) - 4y &= 20 \\ -4y &= 20 \\ y &= -5 \end{aligned} \qquad \begin{aligned} 5x - 4(0) &= 20 \\ 5x &= 20 \\ x &= 4 \end{aligned}$$

Therefore, the y-intercept is at $(0, -5)$ and the x-intercept is at $(4, 0)$.

Graphing Lines Using Intercepts

A graph of $5x - 4y = 20$ is below.



Graphing Lines Using Intercepts

Note that in the last example, the coefficients A and B were both factors of the constant C.

That is, in the equation $5x - 4y = 20$, both 5 and -4 are factors of 20.

If the standard form of a line has this relationship between the coefficients and the constant term, then the x- and y-intercepts will turn out to be "nice" integral values.

On the other hand, if this relationship is not present, then the intercepts will be rational values and may be more difficult to plot.

In this case, it may be better to rearrange the equation into slope-intercept form instead (although there is no guarantee that this will result in integral points either).

Graphing Lines Using Intercepts

Example

Determine the intercepts of $2x - 5y = 15$, and graph the line.

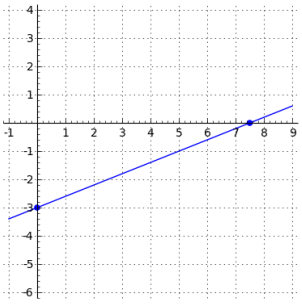
Since 2 is not a factor of 15, the x-intercept will be rational.

$$\begin{aligned} 2(0) - 5y &= 15 \\ -5y &= 15 \\ y &= -3 \end{aligned} \qquad \begin{aligned} 2x - 5(0) &= 15 \\ 2x &= 15 \\ x &= \tfrac{15}{2} \\ x &= 7.5 \end{aligned}$$

Therefore, the y-intercept is at $(0, -3)$, and the x-intercept is at $\left(\frac{15}{2}, 0\right)$ or $(7.5, 0)$.

Graphing Lines Using Intercepts

A graph of $2x - 5y = 15$ is below.



Graphing Lines Using Intercepts

Alternatively, we can graph the line by converting to slope-intercept form.

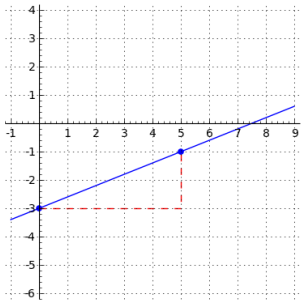
$$\begin{aligned} 2x - 5y &= 15 \\ -5y &= -2x + 15 \\ \frac{-5y}{-5} &= \frac{-2x}{-5} + \frac{15}{-5} \\ y &= \tfrac{2}{5}x - 3 \end{aligned}$$

From the equation, we confirm that the y-intercept is at $(0, -3)$.

From this point, we can use the slope to "rise two and run five" and graph the line.

Graphing Lines Using Intercepts

A graph of $y = \frac{2}{5}x - 3$ is below.



Questions?