MPM1D: Principles of Mathematics

First Differences

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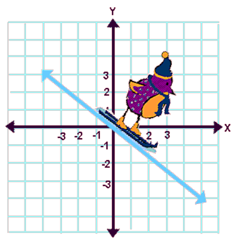
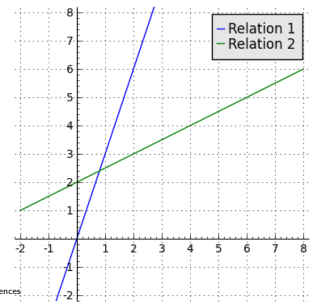


Direct and Partial Variation

Recap

Classify each graph as a direct or partial variation, and determine an equation for each relation.



Direct and Partial Variation

Relation 1 is a straight line passing through the origin, so it is a direct variation.

It passes through the points $(0, 0)$ and $(1, 3)$, so its slope is $\frac{3}{1} = 3$.

Therefore, Relation 1 has the equation $y = 3x$.

Relation 2 is a straight line that does not pass through the origin, so it is a partial variation.

It passes through the points $(0, 2)$ and $(2, 3)$, so its y-intercept is 2 and its slope is $\frac{1}{2}$.

Therefore, Relation 2 has the equation $y = \frac{1}{2}x + 2$.

First Differences

Consider a relation with the following values:

x	0	1	2	3	4
y	7	9	11	13	15

Does this relation represent a direct or partial variation, or neither?

Both relations, whether direct or partial variation, require a constant rate of change (aka slope).

A relation with a constant slope has the same ratio $\frac{\text{rise}}{\text{run}}$ between any two points on its graph.

Thus, one way to identify if a relation is a direct or partial variation is to determine the change in y (the rise) as x changes (the run) by some fixed amount.

First Differences

To measure the change in y each time x increases by 1, we subtract consecutive values of y.

This gives the following table:

x	y	$\Delta 1$
0	7	–
1	9	$9 - 7 = 2$
2	11	$11 - 9 = 2$
3	13	$13 - 11 = 2$
4	15	$15 - 13 = 2$

Note that the value of the first differences is the same each time – that is, the first differences are constant.

This *should* make sense, since different values would indicate a change in the slope of a line, making it non-straight.

First Differences

Linear Relations and First Differences

If a relationship between two variables has constant first differences, then the relationship is *linear*. The graph of such a relation is a straight line, with a slope* equal to the value of the constant first differences. The linear relation may be a direct variation, or a partial variation.

Thus, in the previous example the slope of the line is 2, since it is the value of the constant first differences.

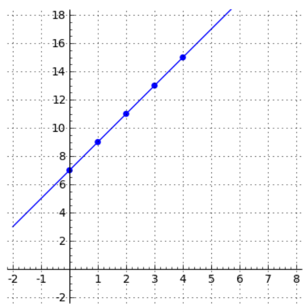
When $x = 0$, $y = 7$, so the relationship is an example of a partial variation, with a constant value of 7.

An equation representing this relation is $y = 2x + 7$.

* *Technically, the value of the first differences is equal to the "rise", but if x increases by 1, it is also equal to the slope.*

First Differences

A graph of the relation confirms that it is linear, and a partial variation.



First Differences

Example

Classify the relation below as linear or non-linear.

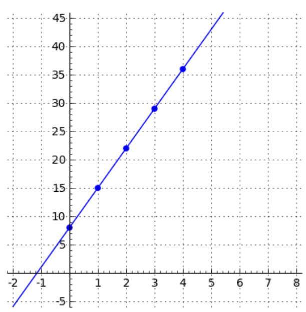
x	0	1	2	3	4
y	8	15	22	29	36

Since x increases by the same amount, find the first differences.

x	y	$\Delta 1$
0	8	–
1	15	$15 - 8 = 7$
2	22	$22 - 15 = 7$
3	29	$29 - 22 = 7$
4	36	$36 - 29 = 7$

First Differences

Since we obtain the same value, 7, for the first differences, the relation is linear.



First Differences

Example

Classify the relation below as linear or non-linear.

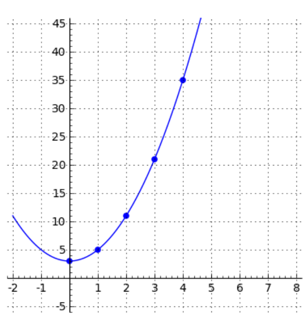
x	0	1	2	3	4
y	3	5	11	21	35

Since x increases by the same amount, find the first differences.

x	y	$\Delta 1$
0	3	–
1	5	$5 - 3 = 2$
2	11	$11 - 5 = 6$
3	21	$21 - 11 = 10$
4	35	$35 - 21 = 14$

First Differences

Since we obtain different first difference values, the relation is non-linear.



First Differences

Example

Classify the relation below as linear or non-linear. If it is linear, determine an equation for the relation.

x	2	4	6	8
y	15	11	7	3

Begin by determining the first differences.

x	y	$\Delta 1$
2	15	–
4	11	$11 - 15 = -4$
6	7	$7 - 11 = -4$
8	3	$3 - 7 = -4$

The first differences are constant, indicating a linear relation.

First Differences

This scenario is different for a few reasons.

The value of the finite differences is negative, indicating that the relation has a negative slope.

While x increases by the same amount, it increases by 2 each time instead of 1. This means the "run" of our slope will be 2, rather than 1.

Since the value of the finite differences is equal to the "rise" of our slope, the slope is $-\frac{4}{2} = -2$.

The table of values begins at $x = 2$, rather than $x = 0$, so we will need to work backward to find the constant value if it is a partial variation.

First Differences

The y-values of the relation decrease by 4 for each increase of 2 in x.

If we go backward and *decrease* x by 2, we need to *increase* y by 4 instead.

Thus, when $x = 0$, $y = 15 + 4 = 19$.

Therefore, an equation for the linear relation is $y = -4x + 19$.

Questions?