Exponent Laws With Numerical Bases

Recap

Simplify, then evaluate, \((\frac{5^3 \times 5^9}{5^4})^3\).

Use the product, quotient and power of a power rules.

\[
\left(\frac{5^3 \times 5^9}{5^4}\right)^3 = \left(\frac{5^{12}}{5^4}\right)^3 = \left(5^8\right)^3 = 5^{24}
\]

Exponent Laws With Variable Bases

In all of the previous examples we have dealt with powers with a numerical base, like \(2^3\).

An algebraic power may use a variable base instead, and may include a leading numerical value known as a coefficient.

For example, the power \(3x^5\) has a variable \(x\), an exponent 5, and a coefficient 3.

The power \(xy^3\) has two variables, \(x\) and \(y\), with exponents 4 and 3 respectively.

Since no coefficient is specified, it is assumed that it has a value of 1.

Example

Simplify \(x^8 \cdot x^{11}\).

Using the product rule, \(x^8 \cdot x^{11} = x^{8+11} = x^{19}\).

Example

Simplify \((x^5)^6\).

Using the power of a power rule, \((x^5)^6 = x^{5 \cdot 6} = x^{30}\).
Exponent Laws With Variable Bases

Example
Simplify \( \left( \frac{x^7 \cdot x^9}{x^{12}} \right)^8 \)

Use all three exponent laws.

\[
\left( \frac{x^7 \cdot x^9}{x^{12}} \right)^8 = \left( \frac{x^{16}}{x^{12}} \right)^8 = \left( x^4 \right)^8 = x^{32}
\]

Exponent Laws With Variable Bases

Consider the expression \( 2x^3 \cdot 4x^2 \).

Since multiplication is commutative, we can rearrange the values.

\[
2x^3 \cdot 4x^2 = 2 \cdot 4 \cdot x^3 \cdot x^2 = 8x^5
\]

The same is true for division.

\[
\frac{6x^7}{2x^4} = \frac{6}{2} \cdot \frac{x^7}{x^4} = 3x^3
\]

Exponent Laws With Variable Bases

This suggests that the product and quotient rules are valid for powers involving coefficients, provided we multiply or divide the coefficients as necessary.

Product/Quotient Rules w/ Coefficients/Multiple Variables

When using the product/quotient rules with like-base powers involving coefficients, the new coefficient has a value equal to the product/quotient of the given coefficients.

Exponent Laws With Variable Bases

Consider the expression \( (2x)^3 \).

Using the definition of exponentiation, we can rewrite the expression in its longer form.

\[
(2x)^3 = (2x)(2x)(2x) = (2 \cdot 2 \cdot 2)(x \cdot x \cdot x) = 2^3 \cdot x^3 = 8x^3
\]

This suggests the following rule.

Power of a Power Rule w/ Coefficients/Multiple Variables

When using the power of a power rule with a power involving a coefficient, the new coefficient has a value equal to the given coefficient raised to the given power.
Exponent Laws With Variable Bases

Example
Simplify \((8x^3y^5)^2\).

Apply the exponent to the coefficient and to both variables.
\[
(8x^3y^5)^2 = 8^2x^{3\cdot2}y^{5\cdot2} = 64x^6y^{10}
\]

A better method is to simplify first, since the values will be smaller and (possibly) easier to work with.
\[
\left(\frac{6x^5y}{9x^2y^3}\right)^2 = \left(\frac{2x^3}{3y}\right)^2 = \frac{2^2x^{6}}{3^2y^4} = \frac{4x^6}{9y^4}
\]

Both methods lead to the same answer, but the latter is preferred.

Example
Simplify \(\left(\frac{5x^2y^2}{3x^2y^4}\right)^2\).

One method is to apply the exponent to all coefficients and variables, then simplify after.
\[
\left(\frac{5x^2y^2}{3x^2y^4}\right)^2 = \frac{5^2x^{2\cdot2}y^{2\cdot2}}{3^2x^{2\cdot2}y^{4\cdot2}} = \frac{25x^4y^4}{9x^4y^8} = \frac{25}{9} \cdot x^4y^4
\]

Use the power of a power rule, and the quotient rule.
\[
\left(\frac{2x^3}{3x^5}\right)^4 = \frac{2^4x^{3\cdot4}}{3^4x^{5\cdot4}} = \frac{16x^{12}}{81x^{20}} = \frac{16}{81}x^{-8}
\]

Note that the variable \(x\) cancels completely.

Questions?