MPM1D: Principles of Mathematics

Exponent Laws (Variable Bases)

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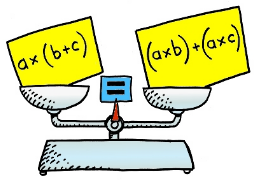

Exponent Laws With Numerical Bases

Recap

Simplify, then evaluate, $\left(\dfrac{5^3 \times 5^9}{5^4}\right)^3$.

Use the product, quotient and power of a power rules.

$$\begin{aligned}\left(\frac{5^3 \times 5^9}{5^4}\right)^3 &= \left(\frac{5^{12}}{5^4}\right)^3 \\ &= \left(5^8\right)^3 \\ &= 5^{24}\end{aligned}$$

Exponent Laws With Variable Bases

In all of the previous examples we have dealt with powers with a numerical base, like 2^3.

An algebraic power may use a *variable* base instead, and may include a leading numerical value known as a *coefficient*.

For example, the power $3x^5$ has a variable x, an exponent 5, and a coefficient 3.

The power x^4y^3 has two variables, x and y, with exponents 4 and 3 respectively.

Since no coefficient is specified, it is assumed that it has a value of 1.

Exponent Laws With Variable Bases

Consider two simple powers with a common variable base, such as x^2 and x^3.

According to the rules of exponentiation, $x^2 = x \cdot x$ and $x^3 = x \cdot x \cdot x$.

If we were to find their product, we would obtain the following.

$$\begin{aligned}x^2 \cdot x^3 &= \overbrace{x \cdot x}^{x^2} \cdot \overbrace{x \cdot x \cdot x}^{x^3} \\ &= x^5\end{aligned}$$

Note that $x^5 = x^{2+3}$, showing that the product rule holds for variable bases.

Exponent Laws With Variable Bases

All three of the previously-explored exponent laws can be used with powers having variable bases.

Exponent Laws

The product rule, quotient rule and power of a power rule can be applied to powers with variable bases.

- product rule: $x^m \cdot x^n = x^{m+n}$
- quotient rule: $\dfrac{x^m}{x^n} = x^{m-n}$
- power of a power rule: $\left(x^m\right)^n = x^{m \cdot n}$

Exponent Laws With Variable Bases

Example

Simplify $x^8 \cdot x^{11}$.

Using the product rule, $x^8 \cdot x^{11} = x^{8+11} = x^{19}$.

Example

Simplify $\left(x^5\right)^6$.

Using the power of a power rule, $\left(x^5\right)^6 = x^{5 \cdot 6} = x^{30}$.

Exponent Laws With Variable Bases

Example

Simplify $\left(\frac{x^7 \cdot x^9}{x^{12}}\right)^8$

Use all three exponent laws.

$$\begin{aligned}\left(\frac{x^7 \cdot x^9}{x^{12}}\right)^8 &= \left(\frac{x^{16}}{x^{12}}\right)^8 \\ &= \left(x^4\right)^8 \\ &= x^{32}\end{aligned}$$

Exponent Laws With Variable Bases

Consider the expression $2x^3 \cdot 4x^2$.

Since multiplication is *commutative*, we can rearrange the values.

$$\begin{aligned}2x^3 \cdot 4x^2 &= 2 \cdot 4 \cdot x^3 \cdot x^2 \\ &= 8x^5\end{aligned}$$

The same is true for division.

$$\begin{aligned}\frac{6x^7}{2x^4} &= \frac{6}{2} \cdot \frac{x^7}{x^4} \\ &= 3x^3\end{aligned}$$

Exponent Laws With Variable Bases

This suggests that the product and quotient rules are valid for powers involving coefficients, provided we multiply or divide the coefficients as necessary.

Product/Quotient Rules w/ Coefficients/Multiple Variables

When using the product/quotient rules with like-base powers involving coefficients, the new coefficient has a value equal to the product/quotient of the given coefficients.

Exponent Laws With Variable Bases

Example

Simplify $5x^2 \cdot 7x^{10}$.

Using the product rule,

$$\begin{aligned}5x^2 \cdot 7x^{10} &= 5 \cdot 7 \cdot x^2 \cdot x^{10} \\ &= 35x^{12}\end{aligned}$$

Example

Simplify $\frac{18x^9}{10x^5}$.

Using the quotient rule,

$$\begin{aligned}\frac{18x^9}{10x^5} &= \frac{18}{10} \cdot \frac{x^9}{x^5} \\ &= \tfrac{9}{5}x^4\end{aligned}$$

Exponent Laws With Variable Bases

Consider the expression $(2x)^3$.

Using the definition of exponentiation, we can rewrite the expression in its longer form.

$$\begin{aligned}(2x)^3 &= (2x)(2x)(2x) \\ &= (2 \cdot 2 \cdot 2)(x \cdot x \cdot x) \\ &= 2^3x^3 \\ &= 8x^3\end{aligned}$$

This suggests the following rule.

Power of a Power Rule w/ Coefficients/Multiple Variables

When using the power of a power rule with a power involving a coefficient, the new coefficient has a value equal to the given coefficient raised to the given power.

Exponent Laws With Variable Bases

Example

Simplify $\left(3x^7\right)^4$.

Apply the exponent to the coefficient and to the variable.

$$\begin{aligned}\left(3x^7\right)^4 &= 3^4x^{7 \cdot 4} \\ &= 81x^{28}\end{aligned}$$

Like the other exponent laws, this can be extended to examples involving more than one variable.

Exponent Laws With Variable Bases

Example

Simplify $(8x^3y^5)^2$.

Apply the exponent to the coefficient and to both variables.

$$\begin{aligned}(8x^3y^5)^2 &= 8^2x^{3\cdot 2}y^{5\cdot 2}\\ &= 64x^6y^{10}\end{aligned}$$

Exponent Laws With Variable Bases

Example

Simplify $\left(\frac{6x^5y}{9x^2y^3}\right)^2$.

One method is to apply the exponent to all coefficients and variables, then simplify after.

$$\begin{aligned}\left(\frac{6x^5y}{9x^2y^3}\right)^2 &= \frac{6^2x^{10}y^2}{9^2x^4y^6}\\ &= \frac{36x^{10}y^2}{81x^4y^6}\\ &= \frac{4x^6}{9y^4}\end{aligned}$$

Exponent Laws With Variable Bases

A better method is to simplify first, since the values will be smaller and (possibly) easier to work with.

$$\begin{aligned}\left(\frac{6x^5y}{9x^2y^3}\right)^2 &= \left(\frac{2x^3}{3y^2}\right)^2\\ &= \frac{2^2x^6}{3^2y^4}\\ &= \frac{4x^6}{9y^4}\end{aligned}$$

Both methods lead to the same answer, but the latter is preferred.

Exponent Laws With Variable Bases

Example

Simplify $\frac{(2x^3)^4}{(3x^6)^2}$.

Use the power of a power rule, and the quotient rule.

$$\begin{aligned}\frac{(2x^3)^4}{(3x^6)^2} &= \frac{2^4x^{3\cdot 4}}{3^2x^{6\cdot 2}}\\ &= \frac{16x^{12}}{9x^{12}}\\ &= \frac{16}{9}\end{aligned}$$

Note that the variable x cancels completely.

Questions?