MPM1D: Principles of Mathematics

Exponent Laws (Numerical Bases)

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Working with Exponents

Recap

Simplify, then evaluate, $2^4 \times 2^6$.

Perform exponentiation before multiplication.

$$\begin{aligned} 2^4 \times 2^6 &= 16 \times 64 \\ &= 1\,024 \end{aligned}$$

Note that $1\,024 = 2^{10}$ and that $2^{4+6} = 2^{10}$.

Exponent Laws

An expression involving some value (or *base*) raised to some exponent is called a *power*.

$$\text{base} \rightarrow \underbrace{2^3}_{\text{power}} \leftarrow \text{exponent}$$

The resulting value is often referred to as a "power of" the base. For example, $2^3 = 8$, so we might say that 8 is a power of 2.

Some people also use the term "power" to refer to the exponent, e.g. "2 to the power of 3", but it is probably more accurate to use the phrase "2 to the exponent 3" instead.

Exponent Laws

In the earlier example, $2^4 \times 2^6$, both powers have the same base.

Their product is also a power of 2, and its exponent is the sum of the two given exponents.

This is easy to see when each power is written in expanded form.

$$\begin{aligned} 2^4 \times 2^6 &= \overbrace{2 \times 2 \times 2 \times 2}^{2^4} \times \overbrace{2 \times 2 \times 2 \times 2 \times 2 \times 2}^{2^6} \\ &= \underbrace{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}_{10 \text{ times}} \\ &= 2^{10} \end{aligned}$$

Exponent Laws

This property is true for the product of any two powers with the same base.

Product Rule for Exponents

For any two powers with the same base, their product is a power with the same base and an exponent equal to the sum of the given exponents.

This allows us to simplify expressions involving multiple powers with the same base.

Exponent Laws

Example

Simplify, then evaluate, $3^2 \times 3^3$.

Since there is a common base of 3, add the exponents.

$$\begin{aligned} 3^2 \times 3^3 &= 3^{2+3} \\ &= 3^5 \\ &= 243 \end{aligned}$$

Exponent Laws

Example

Simplify, then evaluate, $2^8 \times 2^{10}$.

Since there is a common base of 2, add the exponents.

$$\begin{aligned} 2^8 \times 2^{10} &= 2^{8+10} \\ &= 2^{18} \\ &= 262\,144 \end{aligned}$$

Exponent Laws

A similar rule exists when powers with like bases are divided.

Consider the expression $\frac{3^7}{3^5}$. In expanded form, this is

$$\frac{3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3}{3 \times 3 \times 3 \times 3 \times 3}$$

We can cancel each 3 in the denominator with a matching 3 in the numerator.

$$\frac{3 \times 3 \times \cancel{3} \times \cancel{3} \times \cancel{3} \times \cancel{3} \times \cancel{3}}{\cancel{3} \times \cancel{3} \times \cancel{3} \times \cancel{3} \times \cancel{3}}$$

This leaves us with 3×3, or 3^2.

Exponent Laws

Note that $\frac{3^7}{3^5} = 3^2$, and that $3^{7-5} = 3^2$.

Quotient Rule for Exponents

For any two powers with the same base, their quotient is a power with the same base and an exponent equal to the difference of the given exponents.

It is possible to obtain negative exponents using this property. We will talk about these in more detail in later courses.

Exponent Laws

Example

Simplify, then evaluate, $\frac{10^9}{10^6}$.

Since there is a common base of 10, subtract the exponents.

$$\begin{aligned} \frac{10^9}{10^6} &= 10^{9-6} \\ &= 10^3 \\ &= 1\,000 \end{aligned}$$

Exponent Laws

Example

Simplify, then evaluate, $15^4 \div 15^3$.

Since there is a common base of 15, subtract the exponents.

$$\begin{aligned} 15^4 \div 15^3 &= 15^{4-3} \\ &= 15^1 \\ &= 15 \end{aligned}$$

Exponent Laws

What about the expression $\left(5^2\right)^3$?

Expanding this, we get $\left(5^2\right)^3 = 5^2 \times 5^2 \times 5^2$.

Using the product rule, we can add the exponents, since there is a common base of 5.

$$\begin{aligned} 5^2 \times 5^2 \times 5^2 &= 5^{2+2+2} \\ &= 5^6 \end{aligned}$$

Note that $\left(5^2\right)^3 = 5^6$ and that $5^{2\times 3} = 5^6$.

Exponent Laws

This is a third exponent law, applying to a "power of a power".

Power of a Power Rule for Exponents

For any power raised to an exponent, its value is a power with the same base and an exponent equal to the product of the given exponents.

There are other exponent laws in addition to these three, but they will be covered in later courses.

Exponent Laws

Example

Simplify, then evaluate, $\left(2^5\right)^3$.

Multiply the exponents.

$$\begin{aligned}\left(2^5\right)^3 &= 2^{5\times 3}\\ &= 2^{15}\\ &= 32\,768\end{aligned}$$

Exponent Laws

Sometimes, expressions involve multiple exponent laws.

They can usually be applied in any order, but try to keep things simple – make values smaller, rather than larger, and try to cancel things as early as possible.

Exponent Laws

Example

Simplify $\left(2^6 \times 2^3\right)^2$.

Add the exponents inside of the brackets first, then multiply.

$$\begin{aligned}\left(2^6 \times 2^3\right)^2 &= \left(2^{6+3}\right)^2\\ &= \left(2^9\right)^2\\ &= 2^{9\times 2}\\ &= 2^{18}\end{aligned}$$

Exponent Laws

Example

Simplify $\left(\frac{5^{13}}{5^{11}}\right)^4$.

Subtract the exponents inside of the brackets first, then multiply.

$$\begin{aligned}\left(\frac{5^{13}}{5^{11}}\right)^4 &= \left(5^{13-11}\right)^4\\ &= \left(5^2\right)^4\\ &= 5^{2\times 4}\\ &= 5^8\end{aligned}$$

Questions?