Equations of Linear Relations

MPM1D: Principles of Mathematics

Determining Equations of Lines
Part 1: Given the Slope and a Point

J. Garvin

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Slope-Intercept Form of a Line

Recap

State an equation of a line perpendicular to \(4x + 2y = 5\), if it has the same \(y\)-intercept as \(8x - 3y = 15\).

Rearrange the first equation into slope-intercept form.

\[
4x + 2y = 5 \\
2y = -4x + 5 \\
y = -2x + \frac{5}{2}
\]

Since the slope of this line is \(-2\), a perpendicular line will have a slope of \(\frac{1}{2}\).

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Equation of a Line Given the Slope and a Point

Example

Determine an equation of the line with a slope of 3 that passes through the point (4, 7).

Substitute \(m = 3\), \(x = 4\) and \(y = 7\) into \(y = mx + b\) and solve for \(b\).

\[
7 = 3(4) + b \\
7 = 12 + b \\
7 - 12 = b \\
-5 = b
\]

Therefore, an equation of the line is \(y = 3x - 5\).

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Equation of a Line Given the Slope and a Point

Example

Determine an equation of the line with a slope of \(-\frac{1}{2}\) that passes through the point (8, 3).

Substitute \(m = -\frac{1}{2}\), \(x = 8\) and \(y = 3\) into \(y = mx + b\) and solve for \(b\).

\[
3 = -\frac{1}{2}(8) + b \\
3 = -4 + b \\
3 + 4 = b \\
7 = b
\]

Therefore, an equation of the line is \(y = -\frac{1}{2}x + 7\).

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Using the slope formula, the slope of this line is 
$m$. 

Therefore, an equation of the line is $y = 5x + 15$.

Equation of a Line Given the Slope and a Point

Example

Determine an equation of the line with a slope of 5 that has an $x$-intercept at $-3$.

The $x$-intercept has coordinates $(-3, 0)$, so substitute $m = 5$, $x = -3$ and $y = 0$ into $y = mx + b$.

$$0 = 5(-3) + b$$
$$0 = -15 + b$$
$$15 = b$$

Therefore, an equation of the line is $y = 5x + 15$.

Equation of a Line Given the Slope and a Point

Example

Determine an equation of the line that is perpendicular to $3x - 2y = 18$ and has the same $x$-intercept at $y = -3x + 6$.

Rearrange the first equation to determine the slope.

$$3x - 2y = 18$$
$$-2y = -3x + 18$$
$$y = \frac{3}{2}x - 9$$

Since the slope of this line is $\frac{3}{2}$, a perpendicular slope is $-\frac{2}{3}$.

Equation of a Line Given the Slope and a Point

The $x$-intercept of the second equation occurs when $y = 0$.

$$0 = -3x + 6$$
$$3x = 6$$
$$x = 2$$

Thus, the $x$-intercept of the line is $(2, 0)$.

Substitute $m = -\frac{2}{3}$, $x = 2$ and $y = 0$ into $y = mx + b$.

$$0 = -\frac{2}{3}(2) + b$$
$$0 = -\frac{4}{3} + b$$
$$\frac{4}{3} = b$$

Therefore, an equation of the line is $y = -\frac{2}{3}x + \frac{4}{3}$.

Equation of a Line Given the Slope and a Point

Example

Consider a line that passes through two points, $(x, y)$ and $(p, q)$.

Using the slope formula, the slope of this line is $m = \frac{y - q}{x - p}$.

This formula can be rearranged by multiplying both sides by $x - p$.

$$m = \frac{y - q}{x - p}$$
$$m(x - p) = y - q$$

This form is known as point-slope form of a line, and is an alternative to slope-intercept form.

Point-Slope Form of a Line

A line with an equation $y - q = m(x - p)$ is in has a slope of $m$ and passes through point $(p, q)$.

Equation of a Line Given the Slope and a Point

The same equation could be found by substituting $m = -2$, $x = 4$ and $y = 1$ into $y = mx + b$ instead.

$$1 = -2(4) + b$$
$$1 = -8 + b$$
$$1 + 8 = b$$
$$9 = b$$

It is merely a matter of preference which method you use.
Questions?