MPM1D: Principles of Mathematics

Communicating with Algebra

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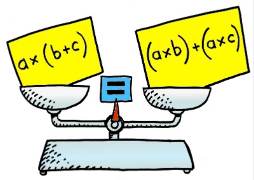

Exponent Laws With Variable Bases

Recap

Simplify, then evaluate, $\frac{2x^3 \times 3x^4}{4x^2}$.

Use the product and quotient rules, watching out for the coefficients.

$$\begin{aligned}\frac{2x^3 \times 3x^4}{4x^2} &= \frac{2 \cdot 3 \cdot x^{3+4}}{4 \cdot x^2} \\ &= \frac{6 \cdot x^7}{4 \cdot x^2} \\ &= \frac{6}{4} \cdot \frac{x^7}{x^2} \\ &= \frac{3}{2} \cdot x^{7-2} \\ &= \tfrac{3}{2}x^5\end{aligned}$$

Classifying Polynomials

A mathematical expression involving one or more powers is a *polynomial*.

Each power makes a *term*. Terms are separated by addition or subtraction.

For example, the expression $3x^2 - 5x + 7$ has three terms.

Polynomials can be classified based on the number of terms they contain, such as a:

- monomial (1 term),
- binomial (2 terms), or
- trinomial (3 terms).

Polynomials with more than 3 terms are generally not given specific names.

Classifying Polynomials

Polynomials can also be classified by the value of the largest exponent in any of its terms. This is called the *degree* of the polynomial.

For example, the degree of the polynomial $4x^3 + 5x - 2$ is 3, since the largest exponent has a value of 3.

Polynomials of the same degree often share similar characteristics, and many are given specific names, such as:

- linear (degree 1),
- quadratic (degree 2),
- cubic (degree 3), and so on.

Classifying Polynomials

Example

Classify the polynomial $5x^2 - 2x + 3$ based on its degree and the number of terms.

The polynomial has degree 2 (quadratic), and is a trinomial (3 terms).

Example

Classify the polynomial $-7x + 5$ based on its degree and the number of terms.

The polynomial has degree 1 (linear), and is a binomial (2 terms).

Classifying Polynomials

Each term in a polynomial may have a *numeric coefficient*, a variable (sometimes called a *literal coefficient*), or both.

A term with no variable component is often called a *constant*, since its value cannot change.

Example

Identify the coefficient and the variable component of the term $-5x^3$.

The coefficient is -5 and the variable component is x^3.

Example

State the value of the constant term in $6x + 5$.

The constant term is 5.

Classifying Polynomials

When a term has more than one variable in it, its degree is the sum of all exponents in the term.

For example, the term $4x^2y$ has a degree of 3, since the sum of the exponents is $2 + 1 = 3$.

Example

State the degree of the polynomial $6x^4y^2 - 10x^5y^3$.

The first term has a degree of $4 + 2 = 6$, while the second term has a degree of $5 + 3 = 8$, so the polynomial has a degree of 8.

Modelling with Polynomials

Throughout this course, we will model various situations using polynomial expressions (usually linear).

It is good form to *define* any variables used in polynomial expressions, so that their purpose is made clear.

These definitions often begin "let x represent..." and are commonly referred to as "let statements".

It is also good practice to give a brief concluding statement that directly answers a particular question.

Modelling with Polynomials

Example

A movie theatre sells two types of tickets: adult tickets for \$12.00/each, and child tickets for \$5.00/each. Create a polynomial expression that represents the total revenue made selling both types of tickets for a movie.

Let a be the number of adult tickets sold, and c the number of child tickets sold.

The total revenue can be modelled by the polynomial $12a + 5c$, where the coefficients represent the price of each type of ticket.

Modelling with Polynomials

Example

Use your expression to determine the revenue generated when 132 adult tickets and 37 child tickets are sold.

Let $a = 132$ and $c = 37$.

$$\begin{aligned} 12a + 5c &= 12(132) + 5(37) \\ &= 1\,769 \end{aligned}$$

Therefore, the revenue generated is \$1 769.00.

Questions?