MPM1D: Principles of Mathematics

Working with Fractions

Part 2: Adding, Subtracting and Negative Fractions

J. Garvin

Multiplying Fractions

Recap

Evaluate $\frac{12}{25} \times \frac{15}{32}$.

Since the GCF of 12 and 32 is 4, and the GCF of 25 and 15 is 5, reduce these values before multiplying.

$$\begin{aligned}\frac{12}{25} \times \frac{15}{32} &= \frac{3}{5} \times \frac{3}{8} \\ &= \frac{9}{40}\end{aligned}$$

Lowest Common Multiple

Consider the two values 6 and 8.

Multiples of 6 are 6, 12, 18, 24, 30, ...

Multiples of 8 are 8, 16, 24, 32, 40, ...

Examining the two lists of multiples, 24 is the first value that appears in *both* lists.

The smallest value that a multiple of two other values is called the *lowest common multiple* (LCM).

This shouldn't be confused with the greatest common factor, which is the *largest* value that divides *into* two other values.

Lowest Common Multiple

Example

Determine the LCM of 8 and 20.

Multiples of 8 are 8, 16, 24, 32, 40, 48, ...

Multiples of 20 are 20, 40, 60, 80, ...

Examining the multiples, the first (and smallest) multiple that is common to both values is 40. Thus, 40 is the LCM of 8 and 20.

Adding/Subtracting Fractions

When adding or subtracting fractions, we need to ensure that the denominators are the same value.

Recall that a fraction represents some part of a whole.

The numerator represents some number of pieces of the whole, while the denominator represents the total number of pieces made from the whole.

When a *common denominator* is used, we can simply count up the total number of pieces via addition or subtraction.

By finding the LCM of two denominators being added or subtracted, we may use it as the common denominator.

Adding/Subtracting Fractions

Example

Evaluate $\frac{3}{16} + \frac{5}{16}$.

Since the denominators are already the same, add the numerators.

$$\begin{aligned}\frac{3}{16} + \frac{5}{16} &= \frac{8}{16} \\ &= \frac{1}{2}\end{aligned}$$

Don't forget to reduce if possible.

Adding/Subtracting Fractions

Example

Evaluate $\frac{7}{10} - \frac{3}{5}$.

In this case, the denominators are different, but since 10 is a multiple of 5 (by a factor of 2), we can easily convert $\frac{3}{5}$ to $\frac{3\times2}{5\times2} = \frac{6}{10}$.

$$\begin{aligned}\frac{7}{10} - \frac{3}{5} &= \frac{7}{10} - \frac{6}{10} \\ &= \frac{1}{10}\end{aligned}$$

Always check to see if one denominator is a multiple of the other first.

Adding/Subtracting Fractions

Example

Evaluate $\frac{3}{8} + \frac{7}{12}$.

Since 12 is not a multiple of 8, we need to find a common denominator before we can add the numerators.

The LCM of 8 and 12 is 24, so we can use this value as the denominator.

Since $3 \times 8 = 24$ and $2 \times 12 = 24$, we must multiply the numerators by these values.

$$\begin{aligned}\frac{3}{8} + \frac{7}{12} &= \frac{9}{24} + \frac{14}{24} \\ &= \frac{23}{24}\end{aligned}$$

Adding/Subtracting Fractions

If we did not identify the LCM of 8 and 12 as 24, it would have been possible to solve the problem by creating a common denominator equal to the product of the two denominators.

$$\begin{aligned}\frac{3}{8} + \frac{7}{12} &= \frac{3\times12}{8\times12} + \frac{7\times8}{12\times8} \\ &= \frac{36}{96} + \frac{56}{96} \\ &= \frac{92}{96} \\ &= \frac{23}{24}\end{aligned}$$

Note that using this approach generally involves larger values that must be reduced later. Using the LCM instead may eliminate this need.

Adding/Subtracting Fractions

Example

Evaluate $\frac{9}{24} - \frac{23}{30}$.

The LCM of 24 and 30 is 120.

$$\begin{aligned}\frac{9}{24} - \frac{23}{30} &= \frac{45}{120} - \frac{92}{120} \\ &= -\frac{47}{120}\end{aligned}$$

Note that, in this case, our answer is negative.

While the fraction may also be written as $\frac{-47}{120}$, it is usually written with the negative sign preceding the entire fraction.

Negative Fractions

Operations with negative fractions are the same as those with negative integers.

- multiplying or dividing two fractions with the same sign produces a positive result.
- multiplying or dividing two fractions with different signs produces a negative result.
- adding a negative fraction is the same as subtracting a positive fraction.
- subtracting a negative fraction is the same as adding a positive fraction.

Negative Fractions

Example

Evaluate $\left(-\frac{3}{8}\right)\left(-\frac{12}{5}\right)$.

Since we are multiplying two negatives, our final answer will be positive.

Reduce the 8 and 12 by a factor of 4, the GCF.

$$\begin{aligned}\left(-\frac{3}{8}\right)\left(-\frac{12}{5}\right) &= \left(\frac{3}{2}\right)\left(\frac{3}{5}\right) \\ &= \frac{9}{10}\end{aligned}$$

Negative Fractions

Example

Evaluate $\frac{5}{6} - \left(-\frac{3}{10}\right)$.

Since we are subtracting a negative, add a positive instead.

Use 30, the LCM of 6 and 10, as a common denominator.

$$\begin{aligned}\frac{5}{6} - \left(-\frac{3}{10}\right) &= \frac{25}{30} + \frac{9}{30} \\ &= \frac{34}{30} \\ &= \frac{17}{15}\end{aligned}$$

Negative Fractions

Example

Evaluate $\frac{2}{5} + \frac{18}{7} \div \left(-\frac{27}{14}\right)$.

Perform the division first, according to the order of operations.

Since we are dividing a positive by a negative, our result will be negative.

$$\begin{aligned}\frac{2}{5} + \frac{18}{7} \div \left(-\frac{27}{14}\right) &= \frac{2}{5} - \left(\frac{18}{7} \times \frac{14}{27}\right) \\ &= \frac{2}{5} - \left(\frac{2}{1} \times \frac{2}{3}\right) \\ &= \frac{2}{5} - \frac{4}{3}\end{aligned}$$

Negative Fractions

Use 15 as a common denominator to add.

$$\begin{aligned}\frac{2}{5} - \frac{4}{3} &= \frac{6}{15} - \frac{20}{15} \\ &= -\frac{14}{15}\end{aligned}$$

Questions?