MHF4U: The Remainder Theorem

1. Identify the divisor, $ax - b$, and the remainder, when each polynomial is divided.
    a. $f(7) = 9$
    b. $g(-2) = 8$
    c. $j\left(\frac{3}{5}\right) = -19$
    d. $k\left(-\frac{4}{9}\right) = \frac{27}{13}$
2. Determine the remainder for each division below. Verify your answers using either long or synthetic division.
    a. $\left(x^3 + 9x^2 - 7x + 1\right) \div (x - 1)$
    b. $\left(4x^4 + 6x^3 - 3x^2 - x + 2\right) \div (x + 2)$
    c. $\left(-3x^5 - 7x^4 + 6x^3 - 3x^2 - 9x - 18\right) \div (x + 3)$
3. Use the Remainder Theorem to determine the remainder when each polynomial is divided by the given binomial.
    a. $f(x) = x^2 - 4x + 7;\ x - 5$
    b. $g(x) = x^3 + 6x - 4;\ x + 3$
    c. $j(x) = 3x^4 + x^2 - 3x + 1;\ x + 2$
    d. $k(x) = 4x^5 - 12x^4 - 4x^3 + 18x^2 - 20x + 9;\ x - 3$
4. Use the Remainder Theorem to determine the remainder when each polynomial is divided by the given binomial.
    a. $f(x) = 2x^3 - 7x^2 + 19x - 17;\ 2x - 1$
    b. $g(x) = 8x^4 - 2x^3 - 16x^2 + 16;\ 4x - 1$
    c. $j(x) = 4x^4 - 6x^3 - 2x^2 + 11x - 19;\ 2x - 3$
    d. $k(x) = 2x^5 + 5x^4 + 6x^2 + 11x - 4;\ 2x + 5$
5. When $x^4 + kx^3 - 2x^2 - 7$ is divided by $x - 2$, the remainder is 33. What is the value of k?
6. For what value of b will $-2x^3 + bx^2 - 10x - 93$ have the same remainder when it is divided by $x + 2$ and by $x - 5$?
7. When $5x^2 + 8x + 11$ is divided by $x - c$, the remainder is 15. Determine the possible value(s) of c.
8. When $2x^3 + ax^2 + bx - 31$ is divided by $x + 2$ the remainder is -7. When the same polynomial is divided by $x - 4$, the remainder is 65. What are the values of a and b?

Solutions

1. a. divisor: $x - 7$, remainder: 9
   b. divisor: $x + 2$, remainder: 8
   c. divisor: $5x - 3$, remainder: -19
   d. divisor: $9x + 4$, remainder: $\frac{27}{13}$
2. a. 4 b. 8 c. -18
3. a. 12 b. -49 c. 59 d. 3
4. a. -9 b. 15 c. -7 d. 6
5. 4
6. 16
7. -2 and $\frac{2}{5}$
8. 2 and -16