

# MHF4U: Polynomial Functions In Factored Form

1. For each polynomial function, state:

- its degree
- its x-intercepts and their orders
- its end behaviour

a.  $f(x) = (x - 2)(x + 5)$

b.  $g(x) = 4(x + 3)(x + 7)^2$

c.  $h(x) = -12(3x - 8)(x + 1)^2(x + 4)^2$

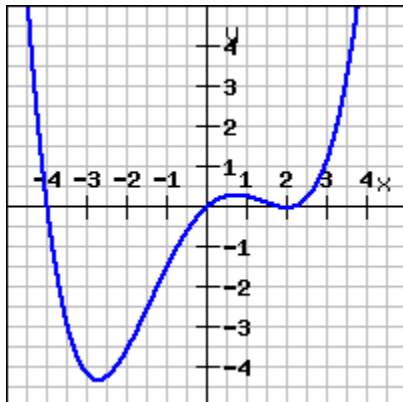
d.  $j(x) = x(x - 11)^2(5x + 2)$

e.  $k(x) = 2(3 - x)(x + 6)^3$

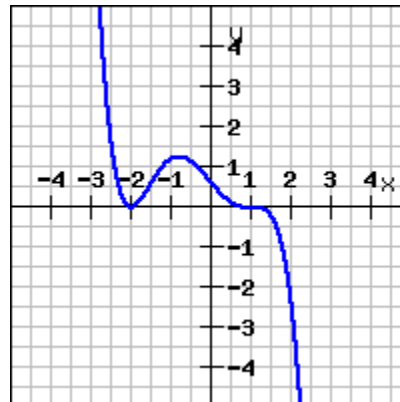
2. Write a possible equation for a polynomial function with the following properties:

- x-intercepts at 3 (order 1) and 5 (order 2); extends from Q2 to Q4
- x-intercepts at 4 (order 2) and -8 (order 2); extends from Q2 to Q1
- x-intercepts at 0 (order 3), 2 (order 1) and -2 (order 1); extends from Q3 to Q1

3. For each graph, state the x-intercepts and the minimum possible order for each.



a.



b.

4. State the x-intercepts and their orders for  $f(x) = (x - 1)(x + 4)^2(x + 6)^3$  and describe the behaviour at each x-intercept.

5. Are all even-degree polynomials even? Justify your answer with an example.

6. State whether each polynomial function is even, odd or neither.

a.  $f(x) = 2(x - 4)(x - 1)(x + 1)(x + 4)$

b.  $g(x) = -4(x - 3)(x + 3)(2x + 7)$

c.  $j(x) = x^5 - 4x^3 + 2x$

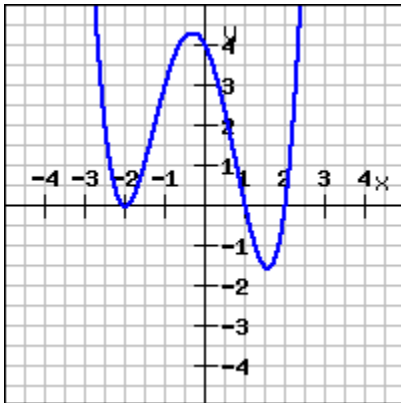
d.  $k(x) = 5x(2 - x)^2(x + 2)^2$

7. Show algebraically that  $p(x) = x^2(x - 5)(x + 5)$  is even.

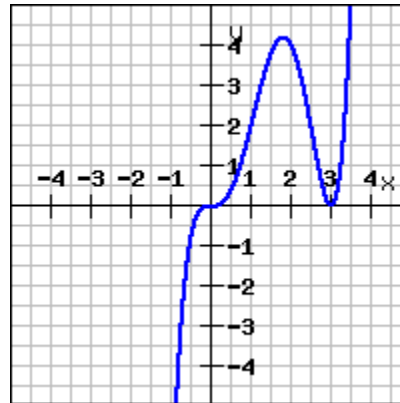
8. Show algebraically that  $q(x) = 7x^5 + 2x^3 - 4x$  is odd.

9. For each graph, state:

- the smallest possible degree of the polynomial function
- the factors of the function
- the intervals where the function is positive or negative



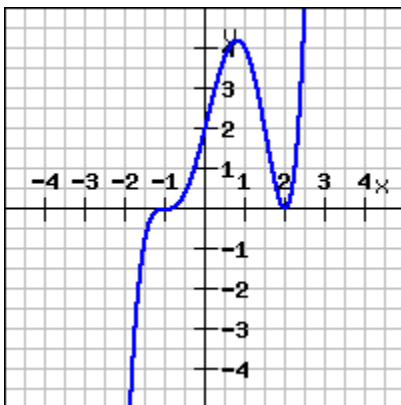
a.



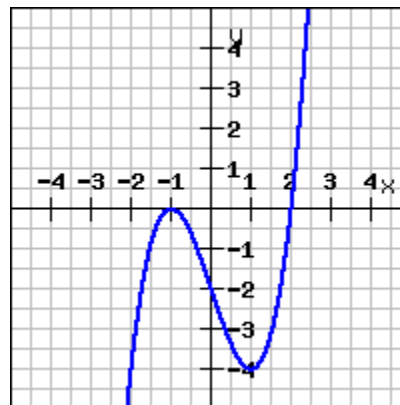
b.

10. Match each graph with its equation.

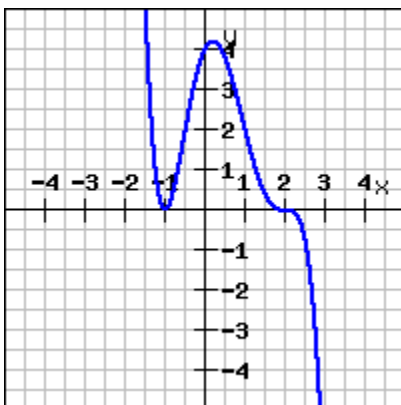
- $f(x) = (x + 1)(x - 2)^2$
- $g(x) = (x - 2)(x + 1)^2$
- $j(x) = \frac{1}{2}(x + 1)^3(x - 2)^2$
- $k(x) = -\frac{1}{2}(x + 1)^2(x - 2)^3$



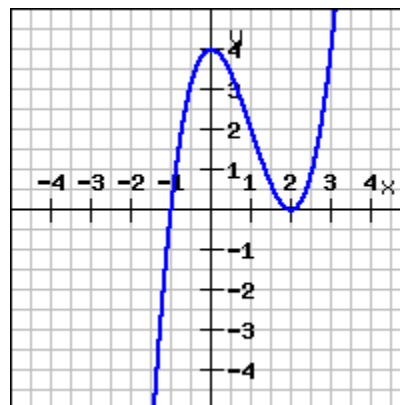
a.



b.



c.



d.

11. For each polynomial function, complete the following chart and sketch a graph. Do not worry about the values of any local minimums or maximums, as we cannot determine them yet.

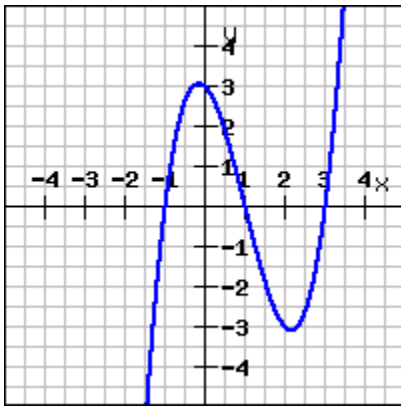
Degree	Sign of Leading Coefficient	End Behaviour	Symmetry	x-Intercepts and Orders	y-Intercept

a.  $f(x) = \frac{1}{4}(x - 1)(x + 2)(x + 4)$

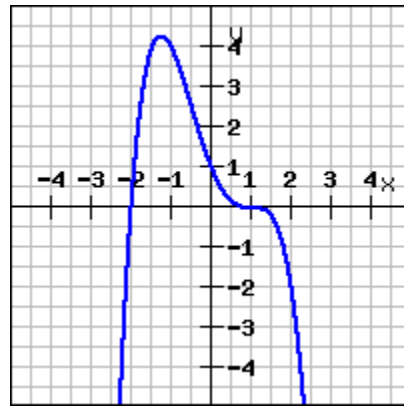
b.  $g(x) = -\frac{1}{9}(x - 3)^2(x + 1)^2$

c.  $h(x) = 12x^3(x - 1)(x + 1)$

12. Determine an equation for each polynomial function.



a.



b.

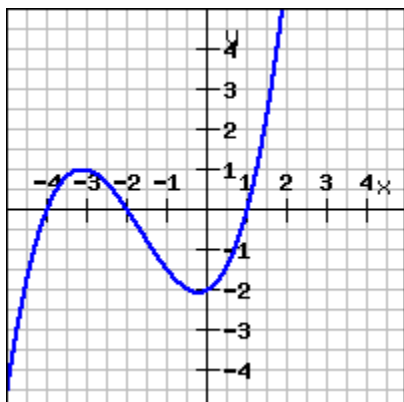
13. Determine an equation for an even polynomial function that has two of its x-intercepts at 2 and -3, if it passes through the point (5, 112).

## Solutions

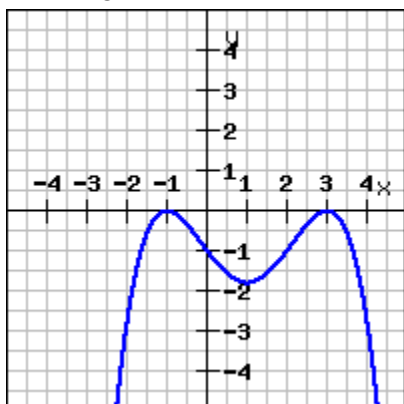
- 2; 2 (ord 1), -5 (ord 1); as  $x \rightarrow -\infty, f(x) \rightarrow \infty$  and as  $x \rightarrow \infty, f(x) \rightarrow \infty$
  - 3; -3 (ord 1), -7 (ord 2); as  $x \rightarrow -\infty, g(x) \rightarrow -\infty$  and as  $x \rightarrow \infty, g(x) \rightarrow \infty$
  - 5;  $\frac{8}{3}$  (ord 1), -1 (ord 2), -4 (ord 2); as  $x \rightarrow -\infty, h(x) \rightarrow \infty$  and as  $x \rightarrow \infty, h(x) \rightarrow -\infty$
  - 4; 0 (ord 1), 11 (ord 2),  $-\frac{2}{5}$  (ord 1); as  $x \rightarrow -\infty, j(x) \rightarrow \infty$  and as  $x \rightarrow \infty, j(x) \rightarrow \infty$
  - 4; 3 (ord 1), -6 (ord 3); as  $x \rightarrow -\infty, k(x) \rightarrow -\infty$  and as  $x \rightarrow \infty, k(x) \rightarrow -\infty$
- equations may vary
- 4 (ord 1), 0 (ord 1), 2 (ord 2)
  - 2 (ord 2), 1 (ord 3)
- 1 (order 1: pass straight through), -4 (order 2: "bounce"), -6 (order 3: pass through "flatter")
- no; answers may vary
- even
  - neither
  - odd
  - odd
- $f(x) = f(-x)$
- $f(x) = -f(-x)$
- 4;  $x + 2, x - 1, x - 2$ ; pos:  $(-\infty, -2) \cup (-2, 1) \cup (1, \infty)$ , neg: (1, 2)
  - 5;  $x, x - 3$ ; pos:  $(0, 3) \cup (3, \infty)$ , neg:  $(-\infty, 0)$

10. a.  $j(x)$  b.  $g(x)$  c.  $k(x)$  d.  $f(x)$

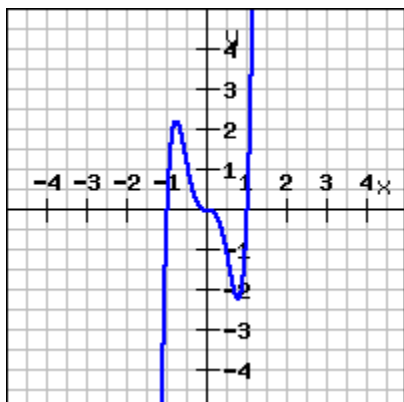
11. a. 3; pos; Q3-Q1; none; 1 (ord 1), -2 (ord 1), -4 (ord 1); -2



b. 4; neg; Q3-Q4; none; -1 (ord 2), 3 (ord 2); -1



c. 5; pos; Q3-Q1; odd; -1 (ord 1), 0 (ord 3), 1 (ord 1); 0



12. a.  $f(x) = (x + 1)(x - 1)(x - 3)$

b.  $g(x) = -\frac{1}{2}(x + 2)(x - 1)^3$

13.  $f(x) = \frac{1}{3}(x - 3)(x - 2)(x + 2)(x + 3)$