MHF4U: Polynomial Division

1. Use long division to determine the quotient and remainder for each division below.
   a. $\left(x^3 + 4x^2 - 6x + 5\right) \div (x + 1)$
   b. $\left(2x^4 - 3x^3 + 2x - 1\right) \div (x - 3)$
   c. $\left(18x^6 - 15x^5 + 3x^4 + 2x^3 - x^2 + 4\right) \div (2x - 1)$
   d. $\left(9x^5 + 12x^4 + 4x^3 - 9x^2 - 18x + 3\right) \div (3x + 2)$
2. Express each division from Q1 in the form $P(x) = (ax - b)Q(x) + R$.
3. Use synthetic division to determine the quotient and remainder for each division below.
   a. $\left(x^2 + 6x - 4\right) \div (x + 2)$
   b. $\left(x^3 + 2x^2 - 5x + 3\right) \div (x - 3)$
   c. $\left(2x^3 + 3x^2 - 8\right) \div (x + 1)$
   d. $\left(2x^4 - 3x^3 + 5x^2 - x + 3\right) \div (x - 2)$
   e. $\left(2x^5 + 5x^4 + 2x^3 - 2x^2 + 5x - 8\right) \div (2x + 1)$
   f. $\left(8x^5 - 6x^4 - 12x^2 + 13x - 1\right) \div (4x - 3)$
4. Express each division from Q3 in the form $P(x) = (ax - b)Q(x) + R$.
5. What is the degree of the quotient for each division below?
   a. $\left(x^5 + 6x^4 - 3x^2 + 1\right) \div (x + 6)$
   b. $\left(3x^4 - 2x^3 + 5x^2 - x + 3\right) \div \left(x^2 + 3x - 4\right)$
   c. $\left(8x^6 - 2x^3 + 3x^2 - 7\right) \div \left(3x^4 + 6x^3 - 2x\right)$
6. A polynomial with degree n is divided by a polynomial with degree k. What degree is the quotient?
7. When divided by $x - 4$, a polynomial has a quotient of $5x^3 - 2x + 1$ and a remainder of 7.
   a. What degree is the polynomial?
   b. What is an equation for the polynomial in standard form?
8. By what polynomial was $P(x) = 6x^3 + 27x^2 + 11x - 9$ divided if it has a quotient of $6x^2 + 3x - 1$ and a remainder of -5?
9. Let $P(x) = 2x^3 + 5x^2 - 39x + 25$. Divide $P(x)$ by $x - 3$ using a method of your choice, note the remainder, then calculate $P(3)$. Divide $P(x)$ by $x + 6$, note the remainder, then calculate $P(-6)$. Make a conjecture about the relationship between the remainder when $P(x)$ is divided by $x - b$, and the value of $P(b)$.
10. Let $P(x) = 4x^4 - 24x^3 + 9x^2 + 61x - 30$. Divide $P(x)$ by $x - 2$ using a method of your choice, and note the remainder. Divide $P(x)$ by $x - 5$ and note the remainder. What does this suggest about the relationship between $P(x)$ and the polynomials $x - 2$ and $x - 5$?

Solutions

1. a. quotient: $x^2 + 3x - 9$, remainder: 14

   b. quotient: $2x^3 + 3x^2 + 9x + 29$, remainder: 86

   c. quotient: $9x^5 - 3x^4 + x^2$, remainder: 4

   d. quotient: $3x^4 + 2x^3 - 3x - 4$, remainder: 11
2. a. $x^3 + 4x^2 - 6x + 5 = (x + 1)\left(x^2 + 3x - 9\right) + 14$

   b. $2x^4 - 3x^3 + 2x - 1 = (x - 3)\left(2x^3 + 3x^2 + 9x + 29\right) + 86$

   c. $18x^6 - 15x^5 + 3x^4 + 2x^3 - x^2 + 4 = (2x - 1)\left(9x^5 - 3x^4 + x^2\right) + 4$

   d. $9x^5 + 12x^4 + 4x^3 - 9x^2 - 18x + 3 = (3x + 2)\left(3x^4 + 2x^3 - 3x - 4\right) + 11$
3. a. quotient: $x + 4$, remainder: -12

   b. quotient: $x^2 + 5x + 10$, remainder: 33

   c. quotient: $2x^2 + x - 1$, remainder: -7

   d. quotient: $2x^3 + x^2 + 7x + 13$, remainder: 29

   e. quotient: $x^4 + 2x^3 - x + 3$, remainder: -11

   f. quotient: $2x^4 - 3x + 1$, remainder: 2
4. a. $x^2 + 6x - 4 = (x + 2)(x + 4) - 12$

   b. $x^3 + 2x^2 - 5x + 3 = (x - 3)\left(x^2 + 5x + 10\right) + 33$

   c. $2x^3 + 3x^2 - 8 = (x + 1)\left(2x^2 + x - 1\right) - 7$

   d. $2x^4 - 3x^3 + 5x^2 - x + 3 = (x - 2)\left(2x^3 + x^2 + 7x + 13\right) + 29$

   e. $2x^5 + 5x^4 + 2x^3 - 2x^2 + 5x - 8 = (2x + 1)\left(x^4 + 2x^3 - x + 3\right) - 11$

   f. $8x^5 - 6x^4 - 12x^2 + 13x - 1 = (4x - 3)\left(2x^4 - 3x + 1\right) + 2$
5. a. 4 b. 2 c. 2
6. $n - k$
7. a. 4 b. $P(x) = 5x^4 - 20x^3 - 2x^2 + 9x + 3$
8. $x + 4$
9. answers may vary
10. answers may vary