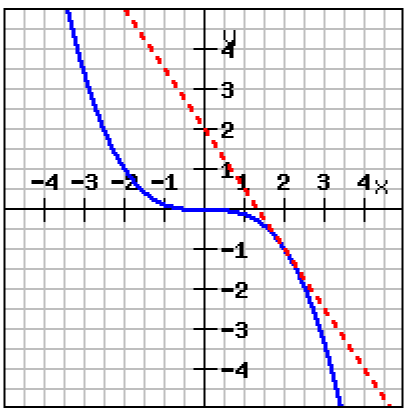
MHF4U: Instantaneous Rates of Change

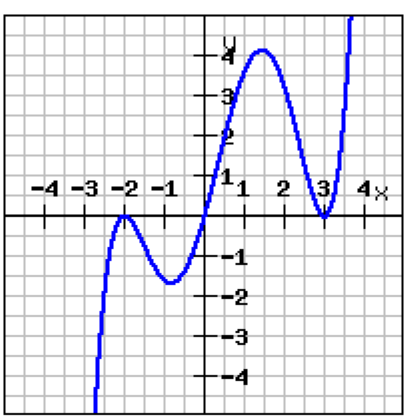
1. Which of the following scenarios describe average rates of change, and which describe instantaneous rates of change?
   a. An airplane travels 3 200 km in 7.5 hours.
   b. Two seconds after jumping, a diver is falling at 16 m/s.
   c. The outside temperature drops 3°C in 45 minutes.
   d. The volume of water in a leaky tank is decreasing by 0.2 L/min at 11:35 a.m.
   e. The marginal revenue when 200 widgets are sold is $4.80 per widget.
2. Graphically, what is the difference between representations of the average rate of change and the instantaneous rate of change?
3. Given the graph below, determine the slope of the tangent to the cubic function at $(2, -1)$.



4. Given the table of values, estimate the instantaneous rate of change when $x = 3$ using two different calculations. How do your answers compare?

x	2	2.5	3	3.5	4	4.5
$f(x)$	10	18	21	23	24	24

5. Given the graph below, state whether the instantaneous rate of change is positive, negative or zero at each value of x.



   a. $x = 0$
   b. $x = 2$
   c. $x = -2$

6. Explain what f, a and h represent in the difference quotient, $f(a) = \frac{f(a+h)-f(a)}{h}$.
7. Estimate instantaneous rate of change for each function at the given value. Note: answers are the *actual* instantaneous rate of change. Your estimates should be close to these values.
    a. $f(x) = 4x^2 - 5x; x = 3$
    b. $g(x) = 2x^3 - 6x + 11; x = 2.5$
    c. $j(x) = 5x^4 + x^2 - 3x; x = - 1.4$
    d. $k(x) = 3x^2 - 9x - 6; x = 2$
8. The height of a ball, *h* metres, *t* seconds after being thrown is given by the equation $h(t) = - 4.9t^2 + 25t + 3$.
    a. estimate the speed of the ball at 2 seconds
    b. estimate the speed of the ball at 4 seconds
    c. what does the sign of your answers tell you about this situation?
9. The difference quotient for some function is $f(5) = \frac{[2(5+h)^3-4(5+h)]-[2(5)^3-4(5)]}{h}$.

    What is the initial value of the interval, and what is the equation of the function?
10. What is the value of the slope of a tangent to a function at a local extremum? Explain how you can estimate if there is a local extremum at some value of x?
11. Use your procedure from Q10 to verify that there is a local extremum for the function $f(x) = 9x^4 + 7x^3 - 10x^2 - 12x$ when $x = \frac{3}{4}$.

Solutions

1. a. average   b. Instantaneous   c. average   d. instantaneous   e. instantaneous
2. slope of a secant = average rate of change, slope of tangent = instantaneous
3. $-\frac{3}{2}$
4. using 2.5: 6; using 3.5: 4; comparisons may vary
5. a. pos   b. neg   c. zero
6. f: some function; a: the beginning value of some interval; h: the width of the interval
7. a. 19   b. 31.5   c. -60.68   d. 3
8. a. 5.4 m/s   b. -14.2 m/s   c. the sign indicates if the ball is moving up (+) or down (-)
9. initial value: 5; equation: $f(x) = 2x^3 - 4x$
10. 0; explanations may vary