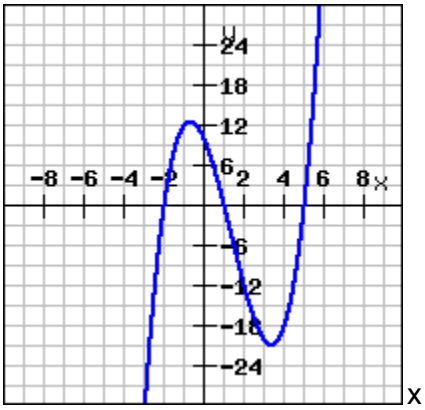
MHF4U: The Factor Theorem

1. Show that each divisor is a factor of the given polynomial. Verify your answers using either long or synthetic division.
   a. $\left(x^3 - 2x^2 - 11x + 12\right) \div (x - 4)$
   b. $\left(2x^4 + 10x^3 - 4x^2 - 17x + 15\right) \div (x + 5)$
   c. $\left(2x^3 - 11x^2 + 2x + 15\right) \div (2x - 3)$
   d. $\left(3x^4 - 13x^3 - 3x^2 + 33x - 20\right) \div (3x + 5)$
2. What values should be considered as potential roots for each polynomial?
   a. $f(x) = x^4 - 2x^2 + 3x - 8$
   b. $g(x) = x^4 + 2x^3 - 5x + 40$
   c. $h(x) = x^5 - 12x^3 + 7x^2 - x + 36$
3. What values should be considered as potential roots for each polynomial?
   a. $p(x) = -2x^3 + 9x^2 + 3$
   b. $q(x) = 4x^5 - 9x^4 + 11x^2 - 5x + 6$
   c. $r(x) = 12x^3 - 5x + 2$
4. Determine whether each binomial is a factor of the given polynomial.
   a. $f(x) = x^3 + 5x^2 + 2x - 8; x - 1$
   b. $g(x) = -2x^3 - 8x^2 + 22x + 60; x + 4$
   c. $j(x) = 6x^4 + 11x^3 + 2x^2 - 5x - 2; 2x - 1$
   d. $k(x) = 4x^5 + x^4 - 8x^3 - 2x^2 + 4x + 1; 4x + 1$
5. Factor each polynomial completely, and sketch a graph of each function.
   a. $f(x) = x^3 - 4x^2 - 7x + 10$
   b. $g(x) = 2x^3 - 11x^2 + 12x + 9$
   c. $h(x) = x^4 - 2x^3 + 27x - 54$
   d. $p(x) = 12x^3 + 20x^2 - x - 6$
   e. $q(x) = 8x^4 + 4x^3 - 18x^2 + 11x - 2$
   f. $r(x) = x^5 + 7x^4 + 19x^3 + 25x^2 + 16x + 4$
6. If $x - 3$ is a factor of $f(x) = 2x^3 - 10x^2 + kx - 15$, determine the value of k.
7. If $2x - 3$ is a factor of $f(x) = 2x^3 + cx^2 - 2x - 24$, determine the value of c.
8. Explain why $f(x) = x^3 - 2x^2 - 5x + 1$ is non-factorable over the integers.
9. Let $f(x) = x^n - 1$ such that $n \in Z$ and $n \geq 1$.
   a. When is $x - 1$ a factor of $f(x)$?
   b. When is $x + 1$ a factor of $f(x)$?
10. Determine values of a and b such that $p(x) = ax^3 - 5bx^2 + 22x + 15$ and $q(x) = 2ax^3 + bx^2 - 49x + 12$ are both divisible by $x - 3$.

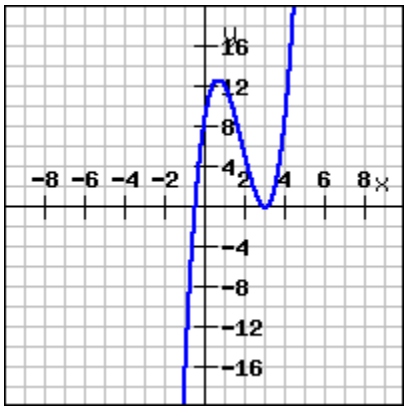
Solutions

2. a. $\pm 1, \pm 2, \pm 4, \pm 8$
   b. $\pm 1, \pm 2, \pm 4, \pm 5, \pm 8, \pm 10, \pm 20, \pm 40$
   c. $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 9, \pm 12, \pm 18, \pm 36$
3. a. $\pm 1, \pm 3, \pm \frac{1}{2}, \pm \frac{3}{2}$
   b. $\pm 1, \pm 2, \pm 3, \pm 6, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{1}{4}, \pm \frac{3}{4}$
   c. $\pm 1, \pm 2, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{2}{3} \pm \frac{1}{4}, \pm \frac{1}{6}, \pm \frac{1}{12}$
4. a. yes b. no c. no d. yes

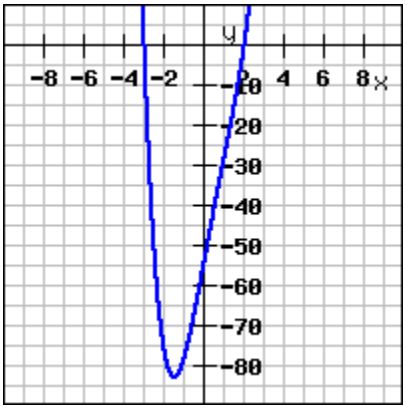
5. a. $f(x) = (x - 1)(x + 2)(x - 5)$



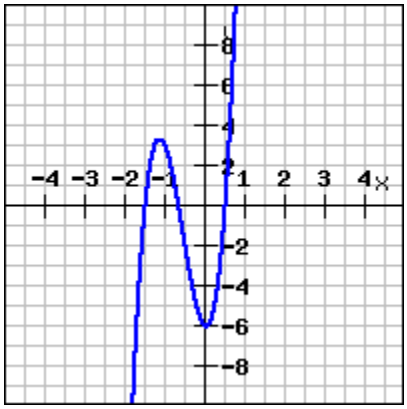
b. $g(x) = (x - 3)^2(2x + 1)$



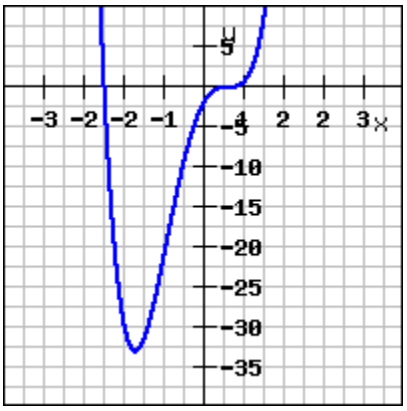
c. $h(x) = (x - 2)(x + 3)(x^2 - 3x + 9)$



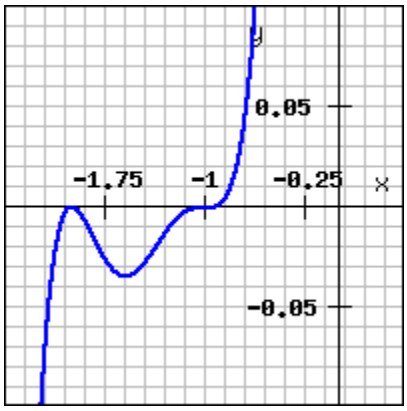
d. $p(x) = (2x - 1)(3x + 2)(2x + 3)$



e. $q(x) = (x + 2)(2x - 1)^3$



f. $r(x) = (x + 2)^2(x + 1)^3$



6. 17
7. 9
8. answers may vary
9. a. always b. when n is even
10. 2 and 3