MHF4U: Descartes' Rule of Signs

1. For each polynomial function, $F(x)$, state:
   - the number of sign changes for $F(x)$
   - the possible number of positive real zeros
   - the number of sign changes for $F(-x)$
   - the possible number of negative real zeros
   - the total possible number of real zeros

   a. $f(x) = x^2 - 5x - 36$

   b. $g(x) = 4x^3 + 9x^2 - x - 3$

   c. $j(x) = -2x^3 + 7x^2 - 4x - 5$

   d. $k(x) = 8x^4 - 6x^3 + 2x^2 + 3x - 6$
2. For each polynomial function, $F(x)$, determine the total possible number of real zeros, then determine the values of the real zeros.

   a. $f(x) = 2x^2 - 7x + 6$

   b. $g(x) = x^3 - 8$

   c. $h(x) = x^3 + 3x^2 - x - 3$

   d. $j(x) = 2x^3 - 7x^2 - 2x - 8$

   e. $k(x) = x^3 - 2x^2 + 3x - 6$

   f. $p(x) = x^4 + x$

   g. $q(x) = 2x^4 - x^3 - 4x^2 + 1$

   h. $r(x) = 2x^4 - 7x^3 + 11x^2 + 9x + 4$
3. The function in Q2h is not factorable using either integer or rational values; however, this alone does not mean that it has no real solutions. Determine the equation of a quadratic function that is not factorable over the integers or rationals but has two real solutions.
4. Use Descartes' Rule of Signs to explain why $f(x) = 3x^2 - 5x - 1$ has exactly two real zeros, one positive and one negative.
5. Explain when a quadratic polynomial of the form $f(x) = ax^2 + bx + c$ will have:
   a. either two or zero positive real zeros
   b. either two or zero negative real zeros
6. A student identifies potential integral zeros of $f(x) = -x^3 + 5x^2 - 2x + 8$ as $\pm 1, \pm 2, \pm 4, \pm 8$. Explain how this list be further reduced.
7. Explain why a cubic polynomial must have at least one real zero:
   a. using a graphical argument
   b. using Descartes' Rule of Signs

Solutions

1. a. sign chg: 1; pos: exactly 1; sign chg: 1: neg: exactly 1; total: 2
   b. sign chg: 1; pos: exactly 1; sign chg: 2: neg: 2 or 0; total: 3 or 1
   c. sign chg: 2; pos: 2 or 0; sign chg: 1: neg: exactly 1; total: 3 or 1
   d. sign chg: 3; pos: 3 or 1; sign chg: 1: neg: exactly 1; total: 4 or 2
2. a. total: 2 or 0; $x = \left\{\frac{3}{2}, 2\right\}$
   b. total: 1; $x = 2$
   c. total: 3 or 1; $x = \{-3, -1, 1\}$
   d. total: 3 or 1; $x = 4$
   e. total: 3 or 1; $x = \left\{-5, 1 + \sqrt{5}, 1 - \sqrt{5}\right\}$
   f. total: 2 (don't forget to factor out x first); $x = \{0, -1\}$
   g. total: 4, 2 or 0; $x = \left\{-1, \frac{1}{2}, \frac{1-\sqrt{5}}{2}, \frac{1+\sqrt{5}}{2}\right\}$
   h. total: 4, 2 or 0; none
3. answers may vary
4. explanations may vary
5. explanations may vary
6. explanations may vary
7. explanations may vary