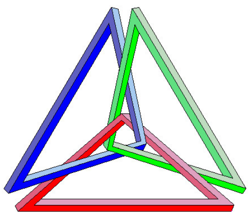
MHF4U: Advanced Functions

Transformations of Sinusoidal Functions

J. Garvin



Transformations of Sinusoidal Functions

The functions $f(x) = \sin x$ and $g(x) = \cos x$ are called *sinusoidal* functions, since they can both be graphed as a sine wave (cosine having the same graph, but translated horizontally).

Since $f(x) = \tan x$ does not share the same general shape, it is a trigonometric function, but not a sinusoidal function.

Sinusoidal functions are often used to model periodic behaviour.

Transformations are applied until the behaviour can be described by a particular function.

Transformations of Sinusoidal Functions

Transformations can be combined to produce more complex sinusoidal functions.

Transformations of Sinusoidal Functions

If $f(x)$ is a sinusoidal function, then $g(x) = af(b(x-c)) + d$ is a sinusoidal function with:

- an amplitude (vertical stretch) of $|a|$,
- a period (horizontal stretch) of $\frac{2\pi}{|b|}$,
- a phase shift (horizontal translation) of c,
- its axis (vertical translation) at $y = d$,
- a minimum value of $d - |a|$,
- a maximum value of $d + |a|$.

Transformations of Sinusoidal Functions

Example

State the amplitude, period, phase shift and location of the axis for the function $f(x) = 4\cos\left(x - \frac{5\pi}{6}\right) - 3$.

- The amplitude is 4.
- The period is 2π.
- The phase shift is $\frac{5\pi}{6}$ to the right.
- The axis is at $y = -3$.

Transformations of Sinusoidal Functions

Example

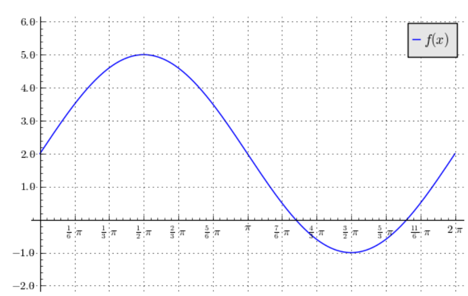
State the amplitude, period, phase shift and location of the axis for the function $f(x) = \frac{2}{5}\sin\left(2x + \frac{\pi}{3}\right)$.

Begin by factoring, $f(x) = \frac{2}{5}\sin\left(2\left[x + \frac{\pi}{6}\right]\right)$.

- The amplitude is $\frac{2}{5}$.
- The period is $\frac{2\pi}{2} = \pi$.
- The phase shift is $\frac{\pi}{6}$ to the left.
- The axis is at $y = 0$.

Transformations of Sinusoidal Functions

Determine an equation for the graph of $f(x)$ below.



Transformations of Sinusoidal Functions

The minimum and maximum values of $f(x)$ are 5 and -1, so the axis is located at $y = \frac{5-1}{2} = 2$.

Since the graph "begins" on $y = 2$, use sine to model the function without a phase shift.

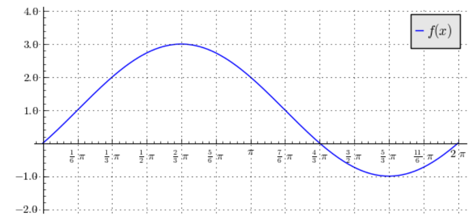
The amplitude is $a = 5 - 2 = 3$.

The period is 2π, so $b = 1$.

An equation, then, is $f(x) = 3\sin x + 2$.

Transformations of Sinusoidal Functions

Determine an equation for the graph of $f(x)$ below.



Transformations of Sinusoidal Functions

The minimum and maximum values of $f(x)$ are 3 and -1, so the axis is located at $y = \frac{3-1}{2} = 1$.

The graph can be modelled using either sine or cosine, both of which require a phase shift.

Using cosine, the "first" maximum value occurs at $\frac{2\pi}{3}$.

The amplitude is $a = 3 - 1 = 2$.

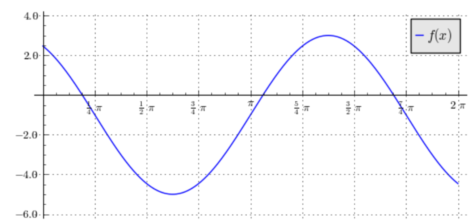
The period is 2π, so $b = 1$.

An equation, then, is $f(x) = 2\cos\left(x - \frac{2\pi}{3}\right) + 1$.

Alternatively, $f(x) = 2\sin\left(x - \frac{\pi}{6}\right) + 1$ also describes the function.

Transformations of Sinusoidal Functions

Determine an equation for the graph of $f(x)$ below.



Transformations of Sinusoidal Functions

The minimum and maximum values of $f(x)$ are 3 and -5, so the axis is located at $y = \frac{3-5}{2} = -1$.

Using sine, the "first" point on the axis occurs at $\frac{\pi}{4}$.

Since the function is decreasing at $\frac{\pi}{4}$, the amplitude is $a = -(3+1) = -4$.

One cycle stretches from $\frac{\pi}{4}$ to $\frac{7\pi}{4}$, so the period is $\frac{3\pi}{2}$, and $b = 2\pi \times \frac{2}{3\pi} = \frac{4}{3}$.

An equation, then, is $f(x) = -4\sin\left(\frac{4}{3}\left(x - \frac{\pi}{4}\right)\right) - 1$.

Alternative equations are $f(x) = 4\sin\left(\frac{4}{3}\left(x - \pi\right)\right) - 1$, $f(x) = 4\cos\left(\frac{4}{3}\left(x - \frac{11\pi}{8}\right)\right) - 1$, etc.

Questions?