MHF4U: Advanced Functions

Transformations of Logarithmic Functions

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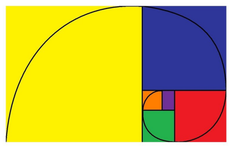
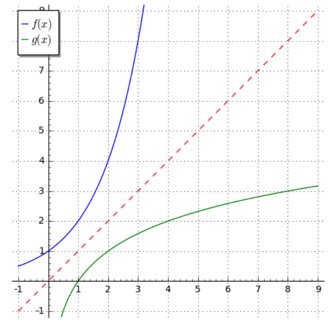


Transformations of Logarithmic Functions

Below are the graphs of $f(x) = 2^x$ and $g(x) = \log_2 x$.



Transformations of Logarithmic Functions

The graph of $g(x) = \log_2 x$ has a vertical asymptote at $x = 0$, corresponding to the vertical asymptote at $y = 0$ for $f(x) = 2^x$.

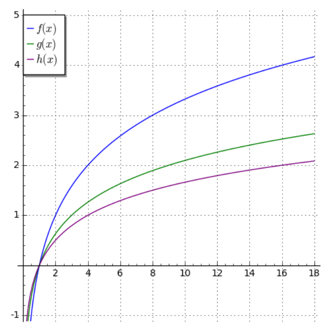
It has an x-intercept at 1, corresponding to the y-intercept of 1 for $f(x) = 2^x$.

Other points on the graph satisfy the equation $y = \log_2 x$. For example, the point $(8, 3)$ is on the graph of $g(x) = \log_2 x$ because $\log_2 8 = 3$.

As a point of similarity, $(3, 8)$ is on $f(x) = 2^x$ because $2^3 = 8$.

Transformations of Logarithmic Functions

Below are the graphs of $f(x) = \log_2 x$, $g(x) = \log_3 x$ and $h(x) = \log_4 x$.



Transformations of Logarithmic Functions

Just like the 2 in $y = 2^x$, the 2 in $y = \log_2 x$ is the base, and is not a transformation.

Like exponential functions, logarithmic functions form a "family" where each function is differentiated by its base.

Without any transformations, they all share two common characteristics:

- a vertical asymptote at $x = 0$, and
- an x-intercept at 1

By keeping track of these things, we can sketch graphs of transformed logarithmic functions fairly quickly.

Transformations of Logarithmic Functions

Transformations of Logarithmic Functions

A function of the form $f(x) = a \log_k(b(x - c)) + d$, where a, b, c and d are real constants, and k is the base of the logarithm, is a transformation of some logarithmic function $g(x) = \log_k x$.

In the form above:

- a is a vertical stretch/compression, and possibly a reflection
- b is a horizontal stretch/compression, and possibly a reflection
- c is a horizontal translation
- d is a vertical translation

Transformations of Logarithmic Functions

Example

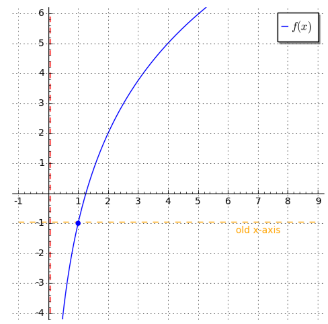
Sketch a graph of $f(x) = 3\log_2 x - 1$.

There are two transformations: a vertical stretch by a factor of 3, and a vertical translation down 1 unit.

Neither of these transformations affects the vertical asymptote. The translation moves the x-intercept down from $(1, 0)$ to $(1, -1)$.

All other points on the graph of $y = \log_2 x$ are now three times as far from the x-axis as they would have been previously.

Transformations of Logarithmic Functions



Transformations of Logarithmic Functions

Example

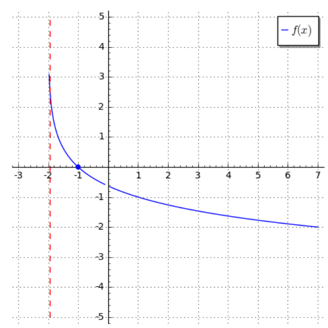
Sketch a graph of $f(x) = -\log_3(x + 2)$.

There are two transformations: a vertical reflection (in the x-axis) and a horizontal translation to the left of two units.

The vertical asymptote moves two units to the left, along with the graph. The x-intercept moves left from $(1, 0)$ to $(-1, 0)$.

Since there is neither a horizontal nor a vertical stretch, all other points preserve their relative distances from the x- and y-axes.

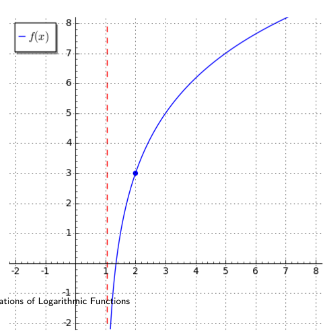
Transformations of Logarithmic Functions



Transformations of Logarithmic Functions

Example

Determine an equation for the transformed graph of $y = \log_2 x$ shown below. The original x-intercept is shown.



Transformations of Logarithmic Functions

Since the vertical asymptote is at $x = 1$, there is a horizontal translation 1 unit to the right.

The original x-intercept has moved from $(1, 0)$ to $(2, 3)$. The change in y-values indicates a vertical translation of 3 units up.

A second, consecutive point on the graph is $(3, 5)$. This point is 1 unit right and 2 units up from the previous point.

On the graph of $y = \log_2 x$, this point would be at $(2, 1)$, which is 1 unit right and 1 unit up. Thus, there is a vertical stretch by a factor of 2.

Therefore, an equation for the function is $f(x) = 2\log_2(x - 1) + 3$.

Questions?