MHF4U: Advanced Functions

Solving Polynomial Equations

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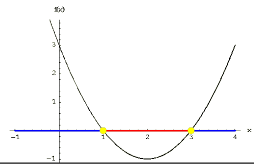


Solving Polynomial Equations

When a polynomial function is assigned a particular value, it is a polynomial equation, such as $2x^3 + 7x^2 + 2x - 3 = 0$.

To solve a polynomial equation like the one above, we must determine any values of x that make the equation true.

To do this, write the equation in factored form, then set each factor equal to zero.

This works, since $(x - r_1) \times (x - r_2) \times \ldots \times (x - r_n) = 0$ implies that one of the factors must be zero.

It is essential to use zero, since it is the only number with this property.

Solving Polynomial Equations

Example

Solve $2x^3 + 7x^2 + 2x - 3 = 0$.

By the RZT, possible factors are ± 1, ± 3, $\pm\frac{1}{2}$ and $\pm\frac{3}{2}$.

Since $2(-1)^3 + 7(-1)^2 + 2(-1) - 3 = 0$, $x + 1$ is a factor.

	2	7	2	−3
−1		−2	−5	3
	2	5	−3	0

Therefore, $(x + 1)(2x^2 + 5x - 3) = 0$.

After decomposition, $(x + 1)(x + 3)(2x - 1) = 0$.

Thus, $x = -1$, $x = -3$ or $x = \frac{1}{2}$.

Solving Polynomial Equations

Example

Solve $x^4 - 9x^3 + 22x^2 - 32 = 0$.

By the IZT, possible factors are ± 1, ± 2, ± 4, ± 8, ± 16 and ± 32.

Since $(2)^4 - 9(2)^3 + 22(2)^2 - 32 = 0$, $x - 2$ is a factor.

	1	−9	22	0	−32
2		2	−14	16	32
	1	−7	8	16	0

Therefore, $(x - 2)(x^3 - 7x^2 + 8x + 16) = 0$.

Solving Polynomial Equations

By the IZT, possible factors of $x^3 - 7x^2 + 8x + 16$ are ± 1, ± 2, ± 4, ± 8 and ± 16.

Since $(-1)^3 - 7(-1)^2 + 8(-1) + 16 = 0$, $x + 1$ is a factor.

	1	−7	8	16
−1		−1	8	−16
	1	−8	16	0

Therefore, $(x - 2)(x + 1)(x^2 - 8x + 16) = 0$.

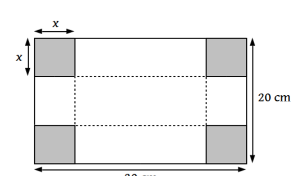
Factoring the perfect square, $(x - 2)(x + 1)(x - 4)^2 = 0$, so the solutions are $x = 4$, $x = 2$ and $x = -1$.

Solving Polynomial Equations

Example

An open-topped box is made by cutting congruent squares from each of the four corners of a 20 cm by 30 cm piece of cardboard and folding up the sides. Determine the side length of the squares that must be cut to create a box with a volume of 1 008 cm^3.

Let x represent the side length of the square, as shown below.



Solving Polynomial Equations

A function to model the volume of the box is $V(x) = x(20-2x)(30-2x)$, or $V(x) = 4x^3 - 100x^2 + 600x$.

An equation is $4x^3 - 100x^2 + 600x = 1\,008$, or $4x^3 - 100x^2 + 600x - 1\,008 = 0$.

Common factoring out a 4 gives $x^3 - 25x^2 + 150x - 252 = 0$.

Since $3^3 - 25(3)^2 + 150(3) - 252 = 0$, $x - 3$ is a factor.

	1	−25	150	−252
3		3	−66	252
	1	−22	84	0

Therefore, $(x-3)(x^2 - 22x + 84) = 0$.

Solving Polynomial Equations

Using the quadratic formula on the simple trinomial yields the solutions $x = 11 \pm \sqrt{37}$.

Since $2 \times (11 + \sqrt{37})$ exceeds the width of the cardboard, this solution is rejected.

The remaining solutions, $x = 3$ and $x = 11 - \sqrt{37}$ are both possible candidates, as both would result in a box with a volume of $1\,008$ cm^3.

Questions?