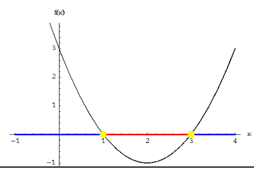
MHF4U: Advanced Functions

Solving Rational Inequalities

Part 2: More Complex Inequalities

J. Garvin



Rational Inequalities

Working with cases can be tedious and prone to error, so an interval table is generally used to solve more complicated rational inequalities.

If one rational function is being compared with another, determining appropriate intervals can be tricky.

In these cases, it is often better to use an *equivalent* function that has some of the same features as the two rational functions.

This equivalent function is created by combining the two rational functions into a single function.

Solving Rational Inequalities

Example

Solve $\frac{3}{x-2} > \frac{1}{x+1}$.

Create a new inequality by subtracting $\frac{1}{x+1}$ from both sides.

$$\frac{3}{x-2} - \frac{1}{x+1} > 0$$
$$\frac{3(x+1)-(x-2)}{(x-2)(x+1)} > 0$$
$$\frac{2x+5}{(x-2)(x+1)} > 0$$

There are vertical asymptotes at $x=-1$ and $x=2$, and an x-intercept at $x=-\frac{5}{2}$.

Solving Rational Inequalities

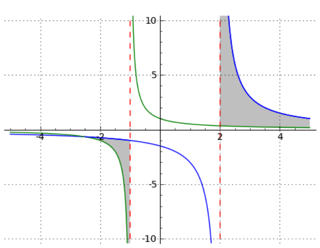
Set up a table with four intervals.

Interval	$(-\infty, -\frac{5}{2})$	$(-\frac{5}{2}, -1)$	$(-1, 2)$	$(2, \infty)$
x	-3	-2	0	3
Sign of $R(x)$	$-$	$+$	$-$	$+$

Since the rational function is positive on two intervals, $\frac{3}{x-2} > \frac{1}{x+1}$ on $(-\frac{5}{2}, -1) \cup (2, \infty)$.

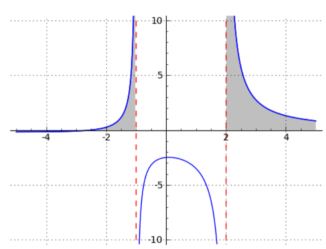
Solving Rational Inequalities

The graph below shows when $\frac{3}{x-2}$ (blue) $> \frac{1}{x+1}$ (green).



Solving Rational Inequalities

The graph below shows when $\frac{2x+5}{(x-2)(x+1)} > 0$.



Solving Rational Inequalities

Example

Solve $\dfrac{2}{x^2+2x-3} < \dfrac{x}{x^2+3x-4}$.

$$\begin{aligned}\frac{2}{x^2+2x-3} - \frac{x}{x^2+3x-4} &< 0\\ \frac{2}{(x+3)(x-1)} - \frac{x}{(x+4)(x-1)} &< 0\\ \frac{2(x+4)-x(x+3)}{(x+3)(x+4)(x-1)} &< 0\\ \frac{-x^2-x+8}{(x+3)(x+4)(x-1)} &< 0\end{aligned}$$

Solving Rational Inequalities

There are vertical asymptotes at $x=-4$, $x=-3$ and $x=1$.
Next, solve $-x^2-x+8=0$ to find the x-intercepts.

$$\begin{aligned}-x^2-x+8&=0\\ x&=\frac{1\pm\sqrt{(-1)^2-4(-1)(8)}}{2(-1)}\\ &=-1\pm\sqrt{33}\end{aligned}$$

There are x-intercepts at $x=-\frac{1+\sqrt{33}}{2}$ (approx. 2.4) and $-\frac{1-\sqrt{33}}{2}$ (approx. -3.4).

Solving Rational Inequalities

Set up a table with six intervals.

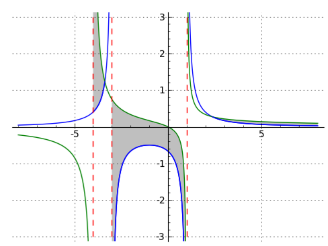
Interval	$(-\infty,-4)$	$\left(-4,-\frac{1-\sqrt{33}}{2}\right)$	$\left(-\frac{1-\sqrt{33}}{2},-3\right)$
x	-5	-3.8	-3.2
Sign of $P(x)$	$+$	$-$	$+$

Interval	$(-3,1)$	$\left(1,-\frac{1+\sqrt{33}}{2}\right)$	$\left(-\frac{1+\sqrt{33}}{2},\infty\right)$
x	0	2	3
Sign of $P(x)$	$-$	$+$	$-$

Therefore, $\dfrac{2}{x^2+2x-3} < \dfrac{x}{x^2+3x-4}$ on $\left(-4,-\frac{1-\sqrt{33}}{2}\right)\cup(-3,1)\cup\left(-\frac{1+\sqrt{33}}{2},\infty\right)$.

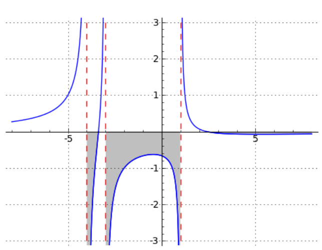
Solving Rational Inequalities

The graph below shows when $\dfrac{2}{x^2+2x-3}$ (blue) $<$ $\dfrac{x}{x^2+3x-4}$ (green).



Solving Rational Inequalities

The graph below shows when $\dfrac{-x^2-x+8}{(x-1)(x+3)(x+4)} < 0$.



Questions?