MHF4U: Advanced Functions

Polynomial and Power Functions

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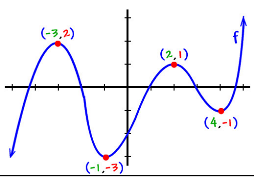


Polynomial Functions

You have seen polynomial functions in previous classes.

Common polynomial functions include constant, linear, quadratic and cubic functions.

A polynomial function has the form

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0$$

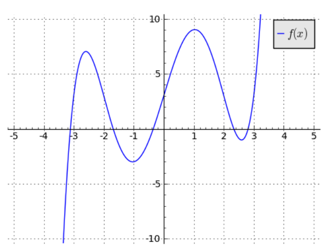
where n is a whole number and a_k is a coefficient.

The *degree* of the polynomial is the exponent of the greatest power.

The *leading coefficient* is a_n, and the constant term is a_0.

Polynomial Functions

Polynomial functions of degree 2 or greater can be drawn using a series of smooth, connected curves, as in the example below.



Classifying Polynomial Functions

Example

Is the function $f(x) = 3x^5 - x^2 + 4x - 1$ a polynomial function?

$f(x)$ is a polynomial function. It has degree 5, a leading coefficient of 3, and a constant term of -1.

Note that there are no x^3 and x^4 terms. This is because the coeffients on these terms, a_3 and a_4, are both zero.

Classifying Polynomial Functions

Example

Is the function $f(x) = 2^x - 5$ a polynomial function?

$f(x)$ is *not* a polynomial function, but an exponential function.

Don't be confused when the variable is the exponent.

Classifying Polynomial Functions

Example

Is the function $f(x) = 3x^{\frac{1}{2}}$ a polynomial function?

$f(x)$ is *not* a polynomial function, but a radical function.

The exponents in a polynomial function are always whole numbers.

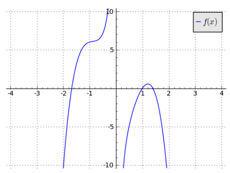
Classifying Polynomial Functions

Example

Is the function $f(x) = -2x^4 + 5x^2 - 3x^{-1}$ a polynomial function?

$f(x)$ is *not* a polynomial function, since the final term has a negative exponent.

A graph of $f(x)$ shows that this is not a polynomial function.



End Behaviour

Recall that a function's *end behaviour* describes how extreme values of the independent variable (x) affect those of the dependent variable ($f(x)$).

We can specify the end behaviour by using the quadrant names, Q1-Q4.

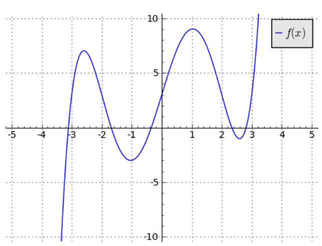
When we want to specify specific values, we say that "as x approaches some value, $f(x)$ approaches some other value."

This is denoted mathematically by "as $x \to A$, $f(x) \to B$" where both A and B are either constants or infinity.

End Behaviour

Example

State the end behaviour of the polynomial function below.

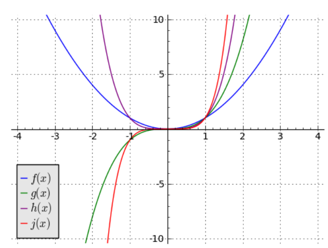


The function extends from Q3-Q1. As $x \to -\infty$, $f(x) \to -\infty$, and as $x \to \infty$, $f(x) \to \infty$.

Power Functions

A polynomial function with the form $f(x) = ax^n$ is called a *power function*.

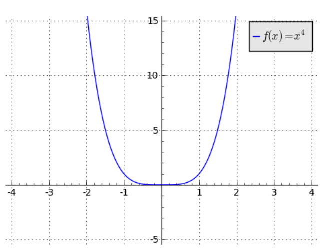
The graphs of $f(x) = x^2$, $g(x) = x^3$, $h(x) = x^4$ and $j(x) = x^5$ are below.



Power Functions

A polynomial function with degree four is called a *quartic* function.

The quartic function $f(x) = x^4$ is similar in shape to the quadratic function $g(x) = x^2$.



Power Functions

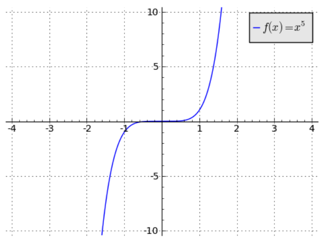
All power functions with even degree have the following characteristics:

- The x- and $f(x)$-intercept is the origin.
- The domain is $(-\infty, \infty)$.
- The range is $[0, \infty)$ if $a > 0$, and $(-\infty, 0]$ if $a < 0$.
- The end behaviour is from Q2-Q1 (as $x \to -\infty$, $f(x) \to \infty$ and as $x \to \infty$, $f(x) \to \infty$) if $a > 0$.
- The end behaviour is from Q3-Q4 (as $x \to -\infty$, $f(x) \to -\infty$ and as $x \to \infty$, $f(x) \to -\infty$) if $a < 0$.
- The graph is symmetric in the $f(x)$-axis.

Power Functions

A polynomial function with degree five is called a *quintic* function.

The quintic function $f(x) = x^5$ is similar in shape to the cubic function $g(x) = x^3$.



Power Functions

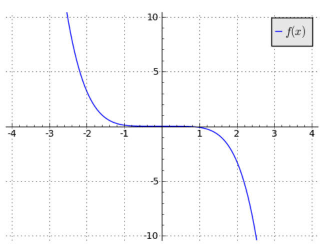
All power functions with odd degree have the following characteristics:

- The x- and $f(x)$-intercept is the origin.
- The domain is $(-\infty, \infty)$.
- The range is $(-\infty, \infty)$.
- The end behaviour is from Q3-Q1 (as $x \to -\infty$, $f(x) \to -\infty$ and as $x \to \infty$, $f(x) \to \infty$) if $a > 0$.
- The end behaviour is from Q2-Q4 (as $x \to -\infty$, $f(x) \to \infty$ and as $x \to \infty$, $f(x) \to -\infty$) if $a < 0$.
- The graph is symmetric about the origin.

Power Functions

Example

Describe the characteristics of a possible power function that could describe the graph below.



Power Functions

The end behaviour is Q2-Q4, so the power function has odd degree.

Given the end behaviour, the leading coefficient of the function is negative.

The graph is symmetric about the origin, further confirming the odd degree.

The function is "flatter" near the origin than the graph of $g(x) = x^3$, implying one of two possible scenarios:

- the degree is 5 or greater.
- there is a vertical compression.

Questions?