MHF4U: Advanced Functions

Logarithms

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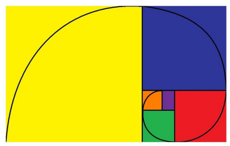

Logarithms

Recall that the inverse of an exponential function, $y = b^x$, swaps the domain with the range to produce $x = b^y$.

To express this relationship, an alternative notation called *logarithmic notation* is used.

Relationship Between Exponents and Logarithms

If $y = b^x$, $b > 0$ and $b \neq 1$, then $x = \log_b y$.

For example, $100 = 10^2$, so $2 = \log_{10} 100$.

Logarithms with a base of 10 are used quite frequently, and are often referred to as *common logarithms*.

$\log_{10} x$ is often abbreviated $\log x$.

Logarithms

Example

Convert $3^4 = 81$ to logarithmic form.

$b = 3$, $x = 4$ and $y = 81$, so $\log_3 81 = 4$.

Example

Convert $5^{-2} = \frac{1}{25}$ to logarithmic form.

$b = 5$, $x = -2$ and $y = \frac{1}{25}$, so $\log_5 \left(\frac{1}{25}\right) = -2$.

Logarithms

Example

Convert $2^{\frac{1}{3}} = \sqrt[3]{2}$ to logarithmic form.

$b = 2$, $x = \sqrt[3]{2}$ and $y = 2^{\frac{1}{3}}$, so $\log_2 \left(\sqrt[3]{2}\right) = \frac{1}{3}$.

Example

Convert $(-2)^{\frac{1}{3}} = \sqrt[3]{-2}$ to logarithmic form.

Since $b < 0$, there is no logarithmic equivalent. This is supported by the fact that the domain of a logarithmic function is $(0, \infty)$.

Logarithms

Example

Convert $\log_7 49 = 2$ to exponential form.

$b = 3$, $x = 2$ and $y = 49$, so $7^2 = 49$.

Example

Convert $\log 1\,000 = 3$ to exponential form.

$b = 10$, $x = 3$ and $y = 1\,000$, so $10^3 = 1\,000$.

Logarithms

To evaluate a logarithmic expression, it is often easier to rewrite it in exponential form.

Example

Evaluate $\log_2 32$.

Since $b = 2$ and $y = 32$, rewrite this as $2^x = 32$.

To determine the value of x, use mental calculations (if the values are small or "obvious") or exponent laws.

$$\begin{aligned} 2^x &= 32 \\ 2^x &= 2^5 \\ x &= 5 \end{aligned}$$

Logarithms

Example

Evaluate $\log_3 \frac{1}{81}$.

Since $b = 3$ and $y = \frac{1}{81}$, rewrite this as $3^x = \frac{1}{81}$.

$$\begin{aligned} 3^x &= \frac{1}{81} \\ 3^x &= \frac{1}{3^4} \\ 3^x &= 3^{-4} \\ x &= -4 \end{aligned}$$

Logarithms

Example

Evaluate $\log 50$.

Since $b = 10$ and $y = 50$, rewrite this as $10^x = 50$.

It is not possible to express 50 as a power of 10, so we cannot use exponent laws at this stage.

You can use the log button on a scientific calculator to obtain an approximate answer, 1.69897.

Logarithms

Example

Evaluate $\log_7 68$.

Since $b = 7$ and $y = 68$, rewrite this as $7^x = 68$.

It is not possible to express 68 as a power of 7, so exponent laws will not help.

While *some* newer scientific calculators will allow a user-specified base other than 10, most do not have this functionality.

Shortly, we will discover a method by which we can "convert" this expression into an equivalent one using base 10 logarithms.

Until then, trial-and-error shows that $7^{2.1684} \approx 68$.

Questions?