MHF4U: Advanced Functions

Linear Inequalities

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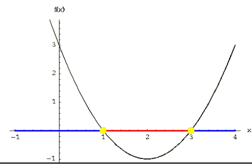


Inequalities

Most of the time, we are interested in determining the value(s) of a variable that result in a particular value.

Other times, however, we are more interested in a *range* of values above or below a fixed value.

An *inequality* is an expression where the equals sign has been replaced by one of the following symbols:

- $>$, "strictly greater than"
- $<$, "strictly less than"
- $\geq$, "greater than or equal to"
- $\leq$, "less than or equal to"
- $\neq$, "not equal to"

Inequalities are often represented on a number line.

Representing Inequalities On a Number Line

Example

Represent $x \leq 3$ on a number line.

A closed dot is used to represent a value that is included in an interval, much like square brackets in interval notation.

Representing Inequalities On a Number Line

Example

Represent $x > -2$ on a number line.

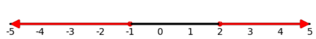
Since the inequality indicates that x is strictly greater than -2, an open dot is used to show that $x = -2$ is not included in the interval.

Representing Inequalities On a Number Line

Example

Represent $\{x \leq -1\} \cup \{x \geq 2\}$ on a number line.

The $\cup$ symbol indicates the *union* of two sets. It is equivalent to "or."



In interval notation, this corresponds to $(-\infty, -1] \cup [2, \infty)$.

Representing Inequalities On a Number Line

Example

Represent $\{x \geq -2\} \cap \{x \leq 3\}$ on a number line.

The $\cap$ symbol represents the *intersection* of two sets. It is equivalent to "and."

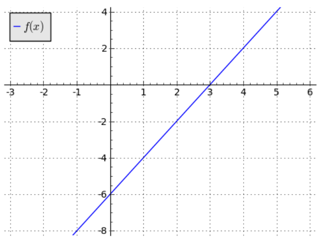
In interval notation, this corresponds to $[-2, 3]$.

Solving Inequalities Graphically

Example

Solve $2x - 6 < 0$.

A graph of $f(x) = 2x - 6$ has a $f(x)$-intercept at -6 and a slope of 2 as shown.



Solving Inequalities Graphically

We are interested in all values of x that result in a negative value of $f(x)$.

All such values are below the x-axis, and occur when $x < 3$.

Therefore, $2x - 6 < 0$ on the for all values of x on $(-\infty, 3)$.

Note that 3 is *not* included in this interval since $2(3) - 6 = 0$, while the inequality specified that $f(x)$ must be strictly less than 0.

Solving Inequalities Algebraically

The rules for solving inequalities are similar to those for solving equations.

Rules for Solving Inequalities

- The same value may be added to, or subtracted from, both sides of an inequality.
- Each side of an inequality may be multiplied, or divided, by the same positive value.
- Each side of an inequality may be multiplied, or divided, by the same negative value *if the inequality is reversed*.
- If each side of an inequality has the same sign, the reciprocal of each side may be taken *if the inequality is reversed*.

Solving Inequalities Algebraically

Example

Solve $4x - 3 \geq 0$.

$$\begin{aligned} 4x - 3 &\geq 0 \\ 4x &\geq 3 \\ x &\geq \frac{3}{4} \end{aligned}$$

Therefore, $4x - 3 \geq 0$ for all values of x on $\left[\frac{3}{4}, \infty\right)$.

Solving Inequalities Algebraically

Example

Solve $-2x + 1 \leq 0$.

$$\begin{aligned} -2x + 1 &\leq 0 \\ -2x &\leq -1 \\ x &\geq \frac{1}{2} \end{aligned}$$

Since both sides of the inequality were divided by -2, the inequality was reversed.

Therefore, $-2x + 1 \leq 0$ for all values of x on $\left[\frac{1}{2}, \infty\right)$.

Solving Inequalities Algebraically

Example

Solve $x - 5 > -2x + 1$.

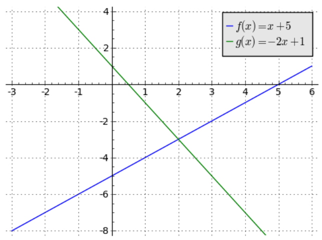
Collect like terms and isolate x.

$$\begin{aligned} x - 5 &> -2x + 1 \\ 3x &> 6 \\ x &> 2 \end{aligned}$$

Therefore, $x - 5 > -2x + 1$ on $(2, \infty)$.

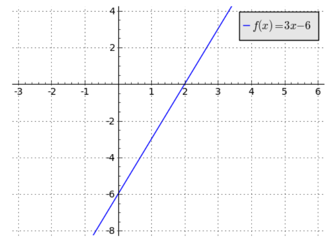
Solving Inequalities Algebraically

The previous example can be illustrated via the graph below.



Solving Inequalities Algebraically

Alternatively, $x - 5 > -2x + 1$ is equivalent to the statement $3x - 6 > 0$, which is also true on $(2, \infty)$.



Questions?