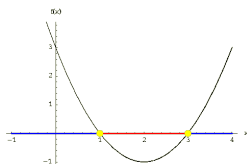


## Families of Polynomial Functions

J. Garvin



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## Families of Polynomial Functions

A *family* of polynomial functions refers to all polynomial functions that share some characteristic.

For example, the family of cubic functions includes all polynomial functions of degree 3.

Often, we are interested in families of polynomial functions of a given degree that have the same  $x$ -intercepts.

These families can be described by the equation  $y = a(x - r_1)(x - r_2) \dots (x - r_n)$ .

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## Families of Polynomial Functions

### Example

Determine an equation for the family of quartic functions that has  $x$ -intercepts at 2, 5 and  $-1$  (order 2).

Since the  $x$ -intercepts are at 2, 5 and  $-1$ , factors are  $x - 2$ ,  $x - 5$  and  $x + 1$ .

Since the intercept at  $x = -1$  has order 2, an equation is  $y = a(x - 2)(x - 5)(x + 1)^2$ .

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## Families of Polynomial Functions

### Example

Determine an equation for a quartic function that has  $x$ -intercepts at 2, 5 and  $-1$  (order 2), if it passes through  $(1, -48)$ .

This function belongs to the family with equation  $y = a(x - 2)(x - 5)(x + 1)^2$ .

Substitute  $x = 1$  and  $y = -48$  and solve for  $a$ .

$$\begin{aligned} -48 &= a(1 - 2)(1 - 5)(1 + 1)^2 \\ -48 &= 16a \\ a &= -3 \end{aligned}$$

An equation is  $f(x) = -3(x - 2)(x - 5)(x + 1)^2$

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## Families of Polynomial Functions

### Example

Determine an equation for the family of cubic functions with  $x$ -intercepts at 2 and  $3 \pm \sqrt{5}$ .

While this family can be expressed as  $y = a(x - 2)(x - 3 - \sqrt{5})(x - 3 + \sqrt{5})$ , it may be easier to work with the equation in standard form.

$$\begin{aligned} y &= a(x - 2)([x - 3] - \sqrt{5})([x - 3] + \sqrt{5}) \\ &= a(x - 2)([x - 3]^2 - 5) \\ &= a(x - 2)(x^2 - 6x + 9 - 5) \\ &= a(x^3 - 8x^2 + 16x - 8) \end{aligned}$$

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## Families of Polynomial Functions

### Example

Determine an equation for the family of cubic functions with  $x$ -intercepts at 2 and  $3 \pm \sqrt{5}$  that passes through  $(5, -6)$ .

Substitute  $x = 5$  and  $y = -6$  into the previous equation in standard form.

$$\begin{aligned} -6 &= a(5^3 - 8(5)^2 + 16(5) - 8) \\ -6 &= -3a \\ a &= 2 \end{aligned}$$

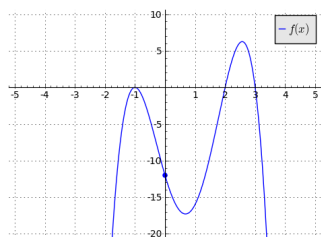
An equation is  $f(x) = 2(x^3 - 8x^2 + 16x - 8)$  or  $f(x) = 2x^3 - 16x^2 + 32x - 16$ .

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## Families of Polynomial Functions

### Example

Determine an equation for quartic function shown below.



## Families of Polynomial Functions

The function has x-intercepts at  $-1$  (order 2),  $2$  and  $3$ .

An equation for the family is  $y = a(x + 1)^2(x - 2)(x - 3)$ .

To find the specific equation, use a clear point on the graph, such as the y-intercept in this case.

$$-12 = a(0 + 1)^2(0 - 2)(0 - 3)$$

$$-12 = 6a$$

$$a = -2$$

An equation is  $f(x) = -2(x + 1)^2(x - 2)(x - 3)$ .

## Questions?

