MHF4U: Advanced Functions

Exponential Functions and Their Inverses

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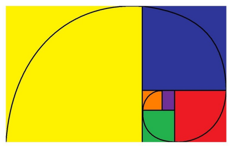


Exponential Functions

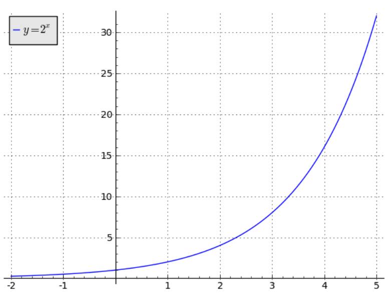
A basic exponential function, without transformations applied to it, has the form $y = b^x$, where b is the *base*.

If $b > 1$, the function is called an *exponential growth* function.

As x increases, y increases rapidly.

Inverse of an Exponential Function

A graph of $y = 2^x$ is below. Note, for example, that when $x = 2$, $y = 2^2 = 4$, and that when $x = -1$, $y = 2^{-1} = \frac{1}{2}$.



Exponential Functions

An exponential function has a repeating pattern in its finite differences.

x	$f(x) = 2^x$	$\Delta 1$	$\Delta 2$	$\Delta 3$
0	1			
1	2	1		
2	4	2	1	
3	8	4	2	1
4	16	8	4	2
5	32	16	8	4

The base of the function is the ratio between any two terms in the finite differences.

Inverse of an Exponential Function

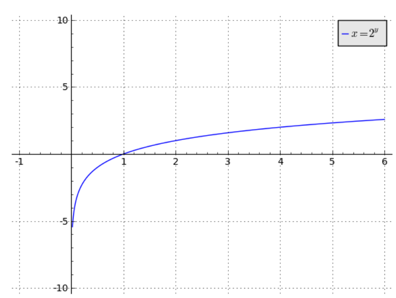
Recall that a function and its inverse are related by switching the independent and dependent variables.

For example, the inverse of the function $y = 2^x$ is $x = 2^y$.

This inverse relation can be graphed either by choosing values for y and substituting them into the equation.

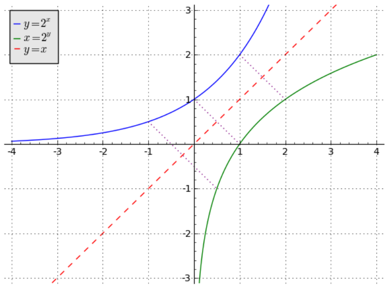
Inverse of an Exponential Function

A graph of $x = 2^y$ is below. Note, for example, that when $y = 2$, $x = 2^2 = 4$, and that when $y = -1$, $x = 2^{-1} = \frac{1}{2}$.



Inverse of an Exponential Function

Graphically, the functions $y = 2^x$ and $x = 2^y$ are reflections in the line $y = x$.



Exponential Decay

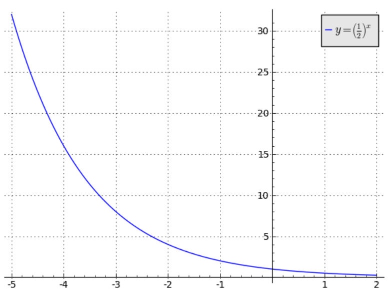
If an exponential function of the form $y = b^x$ has a base where $0 < b < 1$, then the function is an example of *exponential decay*.

Like exponential growth, exponential decay is indicated by a repeating pattern in the finite differences.

x	$f(x) = \left(\frac{1}{2}\right)^x$	$\Delta 1$	$\Delta 2$	$\Delta 3$
0	1			
1	$\frac{1}{2}$	$-\frac{1}{2}$		
2	$\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{2}$	
3	$\frac{1}{8}$	$-\frac{1}{8}$	$\frac{1}{4}$	$-\frac{1}{2}$
4	$\frac{1}{16}$	$-\frac{1}{16}$	$\frac{1}{8}$	$-\frac{1}{4}$

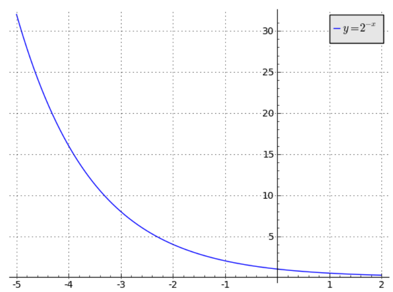
Exponential Decay

The graph below of $y = \left(\frac{1}{2}\right)^x$ shows how exponential decay causes the function to decrease rapidly.



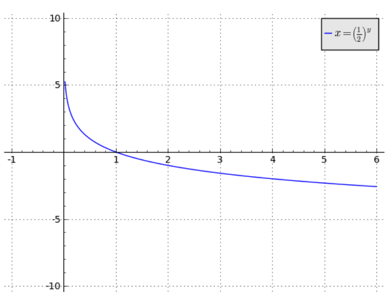
Exponential Decay

Since $2^{-1} = \frac{1}{2}$, the functions $y = \left(\frac{1}{2}\right)^x$ and $y = 2^{-x}$ are equivalent.



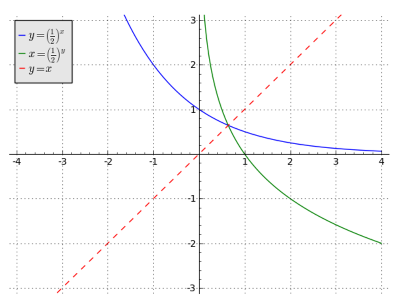
Exponential Decay

Swapping variables, the inverse of $y = \left(\frac{1}{2}\right)^x$ is $x = \left(\frac{1}{2}\right)^y$.



Exponential Decay

$y = \left(\frac{1}{2}\right)^x$ and $x = \left(\frac{1}{2}\right)^y$ are reflections in the line $y = x$.



Properties of Exponential Functions

Exponential functions of the form $y = b^x$ have the following properties:

- y-intercept at 1
- no x-intercepts (HA at $y = 0$)
- growth if $b > 1$, function is always increasing
- decay if $0 < b < 1$, function is always decreasing
- function is positive on $(-\infty, \infty)$

Properties of the Inverses of Exponential Functions

Inverses of exponential functions of the form $x = b^y$ have the following properties:

- no y-intercept (VA at $x = 0$)
- x-intercepts at 1
- growth if $b > 1$, function is always increasing
- decay if $0 < b < 1$, function is always decreasing
- function is positive on $(1, \infty)$ and negative on $(0, 1)$ if $b > 1$; it is positive on $(0, 1)$ and negative on $(1, \infty)$ if $0 < b < 1$.

Questions?