MHF4U: Advanced Functions

Applications of Sinusoidal Functions

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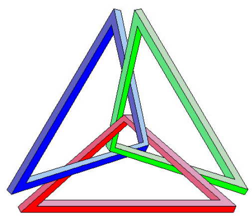

Applications of Sinusoidal Functions

Recall that the period, p, of a sinusoidal function is related to the value of b in $g(x) = af(b(x-c)) + d$ by the equation $b = \frac{2\pi}{p}$.

This is useful for expressing sinusoidal functions in terms of other units, such as time.

For example, imagine a dial that rotates one full turn every 5 seconds.

To model the position of the dial using a sinusoidal function, we can use a value of $b = \frac{2\pi}{5}$ to represent the period of 5 seconds, rather than using radians.

Applications of Sinusoidal Functions

Example

A ferris wheel has a diameter of 30 m, and completes one full revolution in 120 seconds. Riders board a car at its lowest point, from a platform located 1 m above the ground. Determine an equation to model the height, h metres, of a car after t seconds.

Since the diameter of the ferris wheel is 30 m, the amplitude is $\frac{30}{2} = 15$.

The axis has equation $y = 16$, 15 m above the lowest point.

Since the period of the car is 120 seconds, $b = \frac{2\pi}{120} = \frac{\pi}{60}$.

Riders board at the lowest point, so to avoid a phase shift in the equation, use $h(t) = -cos(t)$ as a base function.

Therefore, an equation is $h(t) = -15\cos\left(\frac{\pi}{60}t\right) + 16$.

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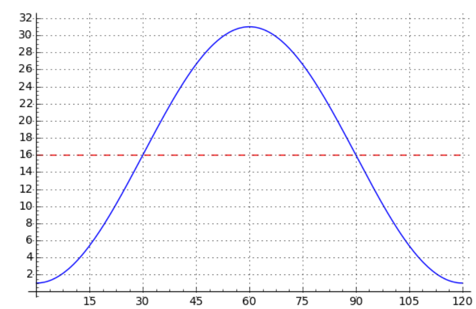

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Example

How high is the car at 40 seconds?

Calculate $h(40)$ to determine its height.

$$\begin{aligned} h(40) &= -15\cos\left(\tfrac{40\pi}{60}\right) + 16 \\ &= 23.5 \end{aligned}$$

At 40 seconds, the car is 23.5 m above the ground.

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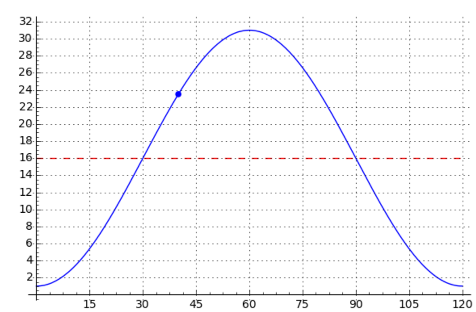

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Example

When is the car at its highest point, and when is its vertical speed the greatest?

Since the function "begins" at its lowest point, the maximum height occurs midway between two lows. One revolution takes 120 seconds, so the maximum is at $\frac{120}{2} = 60$ seconds.

Speed is the change in distance over the change in time, so its greatest speed will occur when the slope of the tangent to the graph is as close to vertical as possible.

There are two such locations for each revolution, both when the car is horizontal with the centre of the ferris wheel.

These two locations correspond to points on the function's axis. These occur at $\frac{60}{2} = 30$ and $60 + 30 = 90$ seconds.

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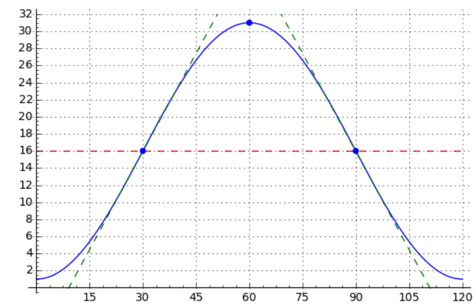

Applications of Sinusoidal Functions

Example

How long is the car 25 m or higher during one revolution?

To find the first time when the car is 25 m above the ground, substitute $h(t) = 25$ and solve for t.

$$\begin{aligned} 25 &= -15\cos\left(\tfrac{\pi}{60}t\right) + 16 \\ -\tfrac{3}{5} &= \cos\left(\tfrac{\pi}{60}t\right) \\ \tfrac{\pi}{60}t &= \cos^{-1}\left(-\tfrac{3}{5}\right) \\ t &= \tfrac{60}{\pi}\cos^{-1}\left(-\tfrac{3}{5}\right) \\ t &\approx 42.29 \end{aligned}$$

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The difference between the first time when the car is 25 above the ground, and the time when the car is at its maximum height, is $60 - 42.29 \approx 17.71$ seconds.

Therefore, since the car travels from 25 m to its maximum height, then back down to 25 m again, the total time will be twice the difference, or $2 \times 17.71 \approx 35.42$ seconds.

Alternatively, the second time the car is 25 high can be found using symmetry at $60 + 17.71 \approx 77.71$ seconds.

The difference between the two times is the same, $77.71 - 42.29 \approx 35.42$ seconds.

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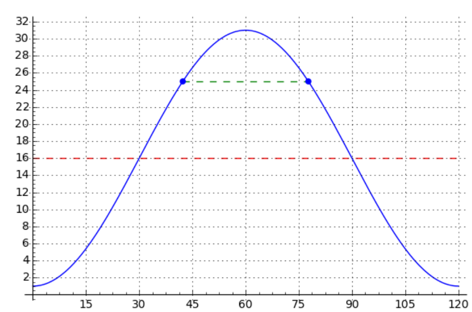

Questions?