MDM4U: Mathematics of Data Management

Applying the Concepts

Problem Solving with Permutations

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Problem Solving with Permutations

Example

In how many ways can eight basketball players align themselves on a bench if either the one centre or both of the two forwards must sit at the leftmost end with the coach?

Case 1: the centre sits next to the coach.

Once the centre is seated, the remaining seven players must be arranged. This can be done in $7! = 5\,040$ ways.

Case 2: the two forwards sit next to the coach.

The two forwards can sit in two ways before the remaining six players must be seated. There are $2 \times 6! = 1\,440$ ways to do this.

Using the RoS, there are $5\,040 + 1\,440 = 6\,480$ ways to seat the players.

Problem Solving with Permutations

Example

With the lights out, Dominique reaches into her closet and randomly grabs two shoes. If there are seven pairs of shoes in the closet, in how many ways can she pick two shoes that do not match?

By the FCP, there are $14 \times 13 = 182$ ways to select a "pair" of shoes, if we differentiate between a left shoe and a right shoe.

There are seven pairs of matched shoes, each of which can be selected in two ways (left-right or right-left), so there are $2 \times 7 = 14$ ways to select a matching pair.

Using an indirect method, there are $182 - 14 = 168$ ways to pick an unmatched pair of shoes.

Problem Solving with Permutations

Example

In how many ways can five boys and five girls line up, if no two boys or girls may be adjacent?

The line must alternate, either BGBGBGBGBG or GBGBGBGBGB.

In the first case, the boys can be arranged in $5!$ ways, as can the girls, for a total of $5! \times 5!$ arrangements.

The second case is symmetric, for an additional $5! \times 5!$ arrangements.

Therefore, the boys and girls can line up in $5! \times 5! + 5! \times 5! = 28\,800$ ways.

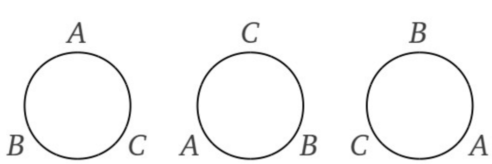
Problem Solving with Permutations

Example

How many ways are there to seat nine people around a circular table?

A round table is different than other shapes, because there is no fixed head-of-table.

For example, the three arrangements below are rotationally identical.



Problem Solving with Permutations

Select one person to represent the head-of-table. This is an arbitrary choice.

This person is now fixed in position, leaving eight people to arrange. This can be done in $8! = 40\,320$ ways.

In general, n people can be seated around a circular table in $(n-1)!$ ways.

Problem Solving with Permutations

Example

How many different ways are there of arranging seven green and eight brown bottles in a row, so that exactly one pair of green bottles is side-by-side?

Place the 8 brown bottles in a row. There are now 9 places to put green bottles (7 between the brown bottles and 2 at the ends).

____ B ____ B ____ B ____ B ____ B ____ B ____ B ____ B ____

Select 1 of the 9 spaces for the pair of green bottles. There are 9 ways of doing this, such as:

____ B ____ B ____ B ____ B ____ B ____ B GG B ____ B ____

Problem Solving with Permutations

In the remaining 8 spaces, we must arrange 5 identical green bottles while leaving three spaces empty (also identical).

There are $\frac{8!}{5!3!} = 56$ possible configurations.

Therefore, the number of ways of placing the bottles so that exactly one pair of green bottles is together is $9 \times 56 = 504$.

Questions?