MDM4U: Mathematics of Data Management

Probabilities and Counting

Determining Probabilities Using Counting Techniques

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Probability Using Counting Techniques

Some probabilities can be calculated using the three main counting techniques covered earlier.

These include:

- Fundamental Counting Principle
- Permutations
- Combinations

Fundamental Counting Principle

Many probability calculations rely on computing the number of choices for one item, followed by the number of choices for a second, and so on.

The FCP is a useful tool for these situations.

Fundamental Counting Principle

Example

What is the probability that a randomly-selected Canadian postal code contains no two identical letters or digits?

Solution: Let C be the event *a postal code with unique characters is selected*.

$n(S) = 26 \times 10 \times 26 \times 10 \times 26 \times 10 = 17\,576\,000$.

$n(C) = 26 \times 10 \times 25 \times 9 \times 24 \times 8 = 11\,232\,000$.

$P(C) = \frac{n(C)}{n(S)} = \frac{11\,232\,000}{17\,576\,000} = \frac{108}{169}$.

Fundamental Counting Principle

Your Turn

What is the probability that a random 4-digit odd integer is a palindrome?

Solution: Let F be the event *a 4-digit odd palindrome is selected*.

$n(S) = 9 \times 10 \times 10 \times 5 = 4500$.

$n(F) = 1 \times 1 \times 10 \times 5 = 50$.

$P(F) = \frac{n(F)}{n(S)} = \frac{50}{4\,500} = \frac{1}{90}$.

Permutations

Some situations require specific orderings or rankings of items.

Since order is important, permutations are used.

Permutations

Example

Nine students have a race. What is the probability of successfully predicting the three fastest runners?

Solution: Let R be the event *the three fastest runners are predicted*.

$n(S) = {}_9P_3 = \frac{9!}{(9-3)!} = 504$.

$n(R) = 1$ (the order is correct)

$P(R) = \frac{n(R)}{n(S)} = \frac{1}{504}$.

Permutations

Your Turn

Four of seven students randomly line up against a wall. What is the probability that Diana is included in the line-up?

Solution: Let D be the event *Diana is included in the line-up*.

$n(S) = {}_7P_4 = \frac{7!}{(7-4)!} = 840$.

$n(D) = 4 \times {}_6P_3 = \frac{6!}{(6-3)!} = 480$.

$P(D) = \frac{n(D)}{n(S)} = \frac{480}{840} = \frac{4}{7}$.

Combinations

Some probabilities are not influenced by ordering.

For example, a question may ask about the probability of having a three-card hand in cards, or forming a committee.

In these cases, combinations are applicable.

Combinations

Example

What is the probability that a committee of four, selected from six doctors and five nurses, contains exactly two from each group?

Solution: Let C the the event *a valid committee is chosen*.

$n(S) = {}_{11}C_4 = 330$.

$n(C) = {}_6C_2 \times {}_5C_2 = 15 \times 10 = 150$.

$P(C) = \frac{n(C)}{n(S)} = \frac{150}{330} = \frac{5}{11}$.

Combinations

Your Turn

The same six doctors and five nurses will form a second three-member committee, which must contain at least one doctor. What is the probability that Nurse Jane is on this committee?

Solution: For the sample space, eliminate all possible committees composed entirely of nurses. So $n(S) = {}_{11}C_3 - {}_5C_3 = 155$.

Let J be the event *a committee with Jane is chosen*. Then $n(J) = \dfrac{{}_1C_1 \times {}_6C_1 \times {}_9C_1}{2} = 27$.

The reason why we divide by 2 is because we overcount when choosing 1 of 6 doctors, then 1 of the 9 remaining people.

Therefore, $P(C) = \frac{n(C)}{n(S)} = \frac{27}{155}$.

Questions?