MDM4U: Mathematics of Data Management

What Are the Odds That...?

Odds In Favour Of, and Against, Events

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Probability

Recap

Determine the probability of drawing an even-numbered card from a standard deck of 52 cards, and the probability of *not* drawing an even-numbered card.

There are five even-numbered cards (2, 4, 6, 8, 10) in each of four suits (♠, ♡, ♣, ♢), for a total of $4 \times 5 = 20$ even-numbered cards.

The probability of drawing an even-numbered card from the deck is $P(E) = \frac{20}{52} = \frac{5}{13}$.

The probability of *not* drawing an even-numbered card is $P(\overline{E}) = 1 - \frac{5}{13} = \frac{8}{13}$.

Odds

Another way to express probability is by using *odds*.

Odds are commonly used in sports and games of chance to express a player's likelihood of winning or losing.

In the previous example, we could say that the *odds in favour* of drawing an even-numbered card are 5:8. Where do these numbers come from?

Odds In Favour Of an Event

The 5 was the numerator of the probability of drawing an even-numbered card. Since $P(E) = \frac{n(E)}{n(S)}$, the numerator is the number of ways in which the event E can occur.

The 8 was the numerator of the probability of *not* drawing an even-numbered card. Since $P(\overline{E}) = \frac{n(\overline{E})}{n(S)}$, the numerator is the number of ways in which the event E does not occur.

This gives us a definition for the odds in favour of an event E.

Odds In Favour of E

Odds in favour of $E = n(E) : n(\overline{E})$.

Odds In Favour Of an Event

Example

Determine the odds in favour of spinning an odd number on a spinner with five equal sections numbered 1-5.

Solution: Let O be the event *an odd number is spun*.

Then, the probability of spinning an odd number is $P(O) = \frac{3}{5}$, so $n(O) = 3$.

The probability of *not* spinning an odd number is $P(\overline{O}) = \frac{2}{5}$, so $n(\overline{O}) = 2$.

Therefore, the odds in favour of spinning an odd number are $3 : 2$.

Odds In Favour Of an Event

Instead of isolating $n(E)$ and $n(\overline{E})$, it is possible to do the same calculations with $P(E)$ and $P(\overline{E})$ directly.

The probability of spinning an even number is $P(O) = \frac{3}{5}$, and the probability of *not* spinning an odd number is $P(\overline{O}) = \frac{2}{5}$.

Note that $\frac{\frac{3}{5}}{\frac{2}{5}} = \frac{3}{5} \times \frac{5}{2} = \frac{3}{2}$. This gives an alternative definition for the odds in favour of event E.

Odds In Favour of E

Odds in favour of $E = \frac{P(E)}{P(\overline{E})}$.

Odds In Favour Of an Event

Example

A poll suggests that 65% of a town's population will vote in favour of re-electing the current mayor in an election. If a random person is interviewed on the street, what are the odds that (s)he will vote for the mayor?

$$\begin{aligned}\text{odds in favour } E &= \frac{P(E)}{1-P(E)}\\ &= \frac{0.65}{0.35}\end{aligned}$$

The odds in favour of the person voting for the mayor are 65:35.

Odds Against an Event

Sometimes it is preferable to express the *odds against* an event happening.

This is simply the reciprocal (or "reverse") of the odds in favour of E.

Odds Against E

Odds against $E = n(\overline{E}) : n(E)$.
or
Odds against $E = \frac{P(\overline{E})}{P(E)}$.

Odds Against an Event

Example

There is a 30% chance of a reaction when two chemicals are combined. Determine the odds against having a reaction.

Since $P(\overline{E}) = 1 - P(E)$ then,

$$\begin{aligned}\text{odds against } E &= \frac{1-P(E)}{P(E)}\\ &= \frac{0.7}{0.3}\end{aligned}$$

The odds against having a reaction are 7:3.

Odds and Probability

We have converted probabilities into odds. We can also do the opposite, and convert odds into probabilities.

Probability From Odds

If the odds in favour of E are $h : k$, then $P(E) = \frac{h}{h+k}$.

Odds and Probability

To prove this, use the definitions of $P(E)$ and $P(\overline{E})$.

$$\begin{aligned}\text{odds in favour of } E &= \frac{P(E)}{P(\overline{E})}\\ \frac{h}{k} &= \frac{P(E)}{1-P(E)}\\ h - hP(E) &= kP(E)\\ h &= hP(E) + kP(E)\\ h &= (h+k)P(E)\\ P(E) &= \frac{h}{h+k}\end{aligned}$$

Odds and Probability

Example

The odds in favour of snow on New Year's Day are estimated at 20:3. What is the likelihood that it will snow on New Year's Day?

Solution: Let S be the event *it snows on New Year's Day* and use the formula where $h = 20$ and $k = 3$.

Therefore, $P(S) = \frac{20}{20+3} = \frac{20}{23}$.

There is approximately an 87% chance of snow on New Year's Day.

Odds and Probability

Your Turn

The odds in favour of a student earning a credit in this course when (s)he does all assigned homework are 9:1. What is the probability that a student with good homework-completion will *not* earn a credit in this course?

Solution: Let C be the event *a credit is earned*.

Then the probability of earning a credit is $P(C) = \frac{9}{9+1} = \frac{9}{10}$.

Therefore, the probability of *not* earning a credit is $P(\overline{C}) = 1 - \frac{9}{10} = \frac{1}{10}$.

Questions?