MDM4U: Mathematics of Data Management

What Is There In Common?

Mutually Exclusive and Non-Mutually Exclusive Events

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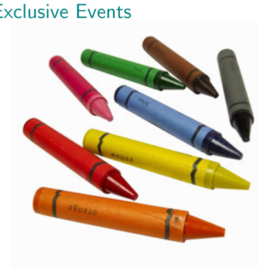


Review

A student has 12 different crayons: 3 shades of red, 2 blues, 4 greens, 2 yellows and 1 purple. How many ways are there of randomly drawing either a red or a green crayon from her pencil case?

There are $n(R) + n(G) = 7$ ways.

Mutually Exclusive Events

Mutually exclusive events are those that have no outcomes in common.

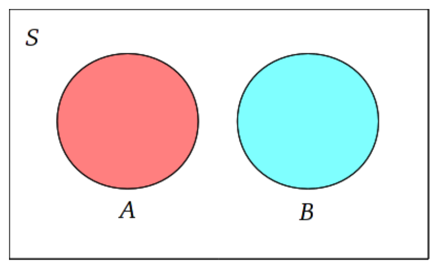
For example, rolling a 6 on a die and rolling a 5 on the same die are mutually exclusive. There can only be one number rolled.

In the review question, a crayon is either red or green. It cannot be both.

Represented using a Venn diagram, these events are *disjoint*.

Mutually Exclusive Events

Two mutually exclusive events, A and B.



Mutually Exclusive Events

Consider the case of rolling either a 6 or a 5 on a die.

Let X be the event *rolling a six*, and F the event *rolling a five*.

$P(X) = \frac{1}{6}$, $P(F) = \frac{1}{6}$, $P(X \text{ or } F) = \frac{2}{6} = \frac{1}{3}$.

Note that $\frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$.

Mutually Exclusive Events

Rule of Sum for Mutually Exclusive Events

If events A and B are mutually exclusive, then $P(A \cup B) = P(A) + P(B)$

Proof

$$\begin{aligned} P(A \cup B) &= \frac{n(A \cup B)}{n(S)} \\ &= \frac{n(A)+n(B)}{n(S)} \\ &= \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)} \\ &= P(A) + P(B) \end{aligned}$$

Mutually Exclusive Events

Example

8 students are waiting in line at the auditorium doors. What is the probability that either Joey or Naomi are first in line?

Let J be the event *Joey is first*, and N the event *Naomi is first*.

Since either Joey or Naomi have a 1 in 8 chance of being first in line, $P(J) = P(N) = \frac{1}{8}$.

Therefore, $P(J \cup N) = P(J) + P(N) = \frac{1}{8} + \frac{1}{8} = \frac{2}{8} = \frac{1}{4}$.

Mutually Exclusive Events

Your Turn

A committee of five is to be formed from six males and eight females. What is the probability that the committee is composed entirely of males, or entirely of females?

Let M be the event *the committee contains all males* and F the event *the committee contains all females*.

There are ${}_{14}C_5$ ways to form the committee with no restrictions, ${}_{6}C_5$ ways to form it from males only, and ${}_{8}C_5$ ways to form it from females only.

So $P(M \cup F) = P(M) + P(F) = \frac{{}_{6}C_5}{{}_{14}C_5} + \frac{{}_{8}C_5}{{}_{14}C_5} = \frac{31}{1001}$.

Non-Mutually Exclusive Events

Some events are not mutually exclusive.

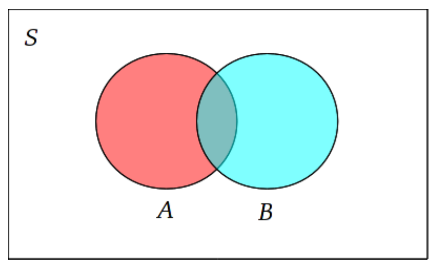
For example, an integer may be divisible by 3, or divisible by 5. These are *not* mutually exclusive events, because it is possible that the integer is divisible by both (e.g. 15).

We need to compensate for this by determining the number of common outcomes that have been overcounted.

These two events are not disjoint on a Venn diagram.

Non-Mutually Exclusive Events

Two mutually exclusive events, A and B.



Non-Mutually Exclusive Events

Principle of Inclusion/Exclusion for Non-Mutually Exclusive Events

If events A and B are mutually exclusive, then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Use the Principle of Inclusion/Exclusion for counting outcomes...

Non-Mutually Exclusive Events

Proof

$$\begin{aligned} P(A \cup B) &= \frac{n(A \cup B)}{n(S)} \\ &= \frac{n(A) + n(B) - n(A \cap B)}{n(S)} \\ &= \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)} - \frac{n(A \cap B)}{n(S)} \\ &= P(A) + P(B) - P(A \cap B) \end{aligned}$$

Non-Mutually Exclusive Events

Example

Determine the probability of rolling an even number, or a number greater than three, on a standard die.

Let E be the event *rolling an even number* and T the event *rolling a number* > 3.

$S = \{1, 2, 3, 4, 5, 6\}$, so $n(S) = 6$. $E = \{2, 4, 6\}$, so $n(E) = 3$. $T = \{4, 5, 6\}$, so $n(T) = 3$.

There are two elements in common, so $(E \cap T) = \{4, 6\}$ and $n(E \cap T) = 2$.

Therefore, the probability of rolling an even number or a number greater than three is $P(E \cup T) = P(E) + P(T) - P(E \cap T) = \frac{3}{6} + \frac{3}{6} - \frac{2}{6} = \frac{2}{3}$.

Non-Mutually Exclusive Events

Your Turn

As a result of a recent survey, a soda manufacturer estimates that 95% of its target population drinks either cola or ginger ale; 85% of the population drinks cola; and 35% drinks ginger ale. What is the likelihood that a randomly selected individual from the target population drinks both?

Let C be the event *a person drinks cola* and G the event *a person drinks ginger ale*.

$$\begin{aligned} P(C \cup G) &= P(C) + P(G) - P(C \cap G) \\ 0.95 &= 0.85 + 0.35 - P(C \cap G) \\ P(C \cap G) &= 0.25 \end{aligned}$$

There is a 25% chance of the individual drinking both.

Questions?