Paths To Treasure

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Cumulative Project (Probability)
MDM4U: Data Management

Overview

In this game, the player rolls a die to determine the direction that he/she travels through a maze. After paying a fixed buy-in, a player continues to travel until he/she reaches one of eight treasure or monster locations. If there is treasure at that location, the player wins a fixed amount of money. If there is a monster, the player loses his/her money.

Materials

The game uses the following materials:

- One game board.
- One six-sided die, with three faces marked “left” and three marked “right”.
- One or more player tokens.

Game Play and Rules

To play the game, a player must pay a buy-in fee of $5. Once this fee has been paid, the player rolls the die and moves through the maze in the direction indicated (left or right). The player keeps rolling until he or she lands on either a treasure chest or a monster square.

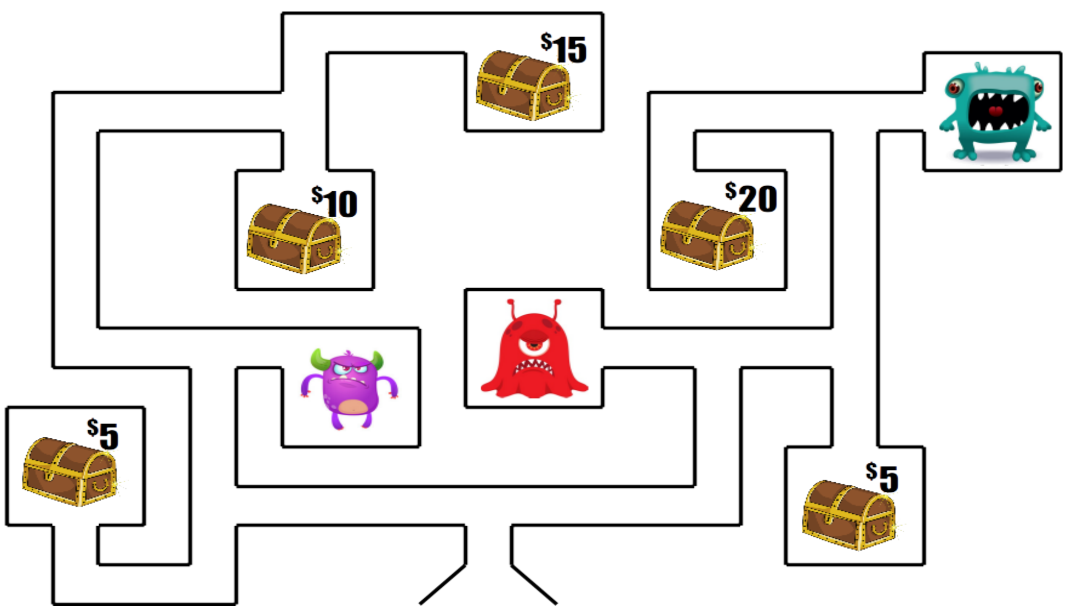

Figure 1: Game Board

If a player lands on a treasure chest, he/she wins the amount of money shown. Five squares contain treasure chests, two of which cause the player to “break even” while three of which result in a monetary gain for the player. If a player lands on a monster square, the player loses his/her buy-in.

Theoretical Probabilities and Expected Value

This is an application of Pascal’s Method, where the probability of landing on each location can be calculated by tracing the probabilities down each path. Since there are three “left” faces, and three “right” faces, on the die, it is equally likely that a player will move left or right. The probabilities of arriving at each final location are shown on the tree diagram below.

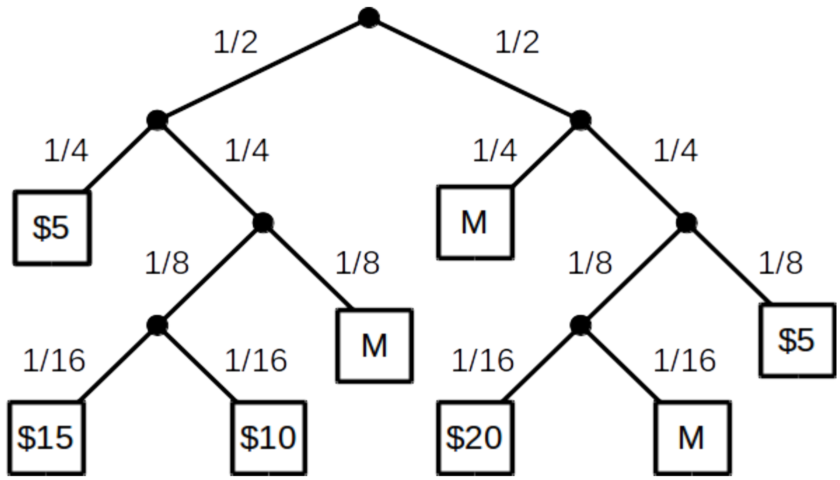

Figure 2: Theoretical Probabilities

There are eight possible sequences of rolls, summarized in the table below.

Sequence	$P(x)$	Payout (x)	$x\,P(x)$
LL	$\frac{1}{4}$	\$5.00	\$1.2500
RL	$\frac{1}{4}$	\$0.00	\$0.0000
LRR	$\frac{1}{8}$	\$0.00	\$0.0000
RRR	$\frac{1}{8}$	\$5.00	\$0.6250
LRLL	$\frac{1}{16}$	\$15.00	\$0.9375
LRLR	$\frac{1}{16}$	\$10.00	\$0.6250
RRLL	$\frac{1}{16}$	\$20.00	\$1.2500
RRLR	$\frac{1}{16}$	\$0.00	\$0.0000
			\$4.6875

Table 1: Expected Payout per Play

All possibilities are accounted for, as the sum of the probabilities is 1.

$$\frac{1}{4}+\frac{1}{4}+\frac{1}{8}+\frac{1}{8}+\frac{1}{16}+\frac{1}{16}+\frac{1}{16}+\frac{1}{16}=1$$

The expected payout is the sum of the net payouts, minus the $5 buy-in.

$$E(X)=4.6875-5.00\approx-0.31$$

A player can be expected to lose approximately 31 cents per play. This shows that the game is not fair.

Methodology

The game was designed to appear easy to win, when it reality, a player has a greater chance of losing his/her money. Since there are a greater number of squares on which a player can win or “break even”, it appears as if the game is biased in the player’s favour. Furthermore, the simple act of tossing a die with two possible outcomes is fairly deceptive – a player might be suspicious if they had to roll a 6 from six possible options, but in this case, a player might feel lucky enough to roll the “left” or “right” that he/she is hoping for.

Payouts were chosen to ensure a biased outcome in favour of the House. They are fairly high (up to four times the buy-in) to encourage players to play. A spreadsheet was used to modify the payouts until an appropriate expected value was found. For example, using a maximum payout of $25 instead of $20 results in a fair game, which does not meet the goals of this project. Other options, such as including more losing squares, resulted in a net loss that was too great, possibly discouraging further play.

Conclusions

An expected loss of 31 cents per play results in a game that is biased against the player. On average, the player stands to lose this game, even though the number of winning scenarios exceeds the number of losing scenarios. By carefully assigning payouts with probabilities determined using Pascal’s Method, the House stands to win in the long run, while not appearing overly biased.