MCV4U: Calculus & Vectors

Vector Basics

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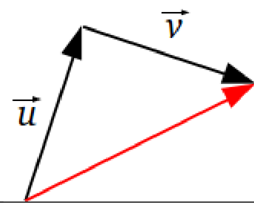


Vectors vs. Scalars

A *vector* is a quantity with both *magnitude* and *direction*.

A *scalar* is a quantity that has only *magnitude*.

For example, speed is a scalar (80 km/h), while velocity is a vector (75 km/h NE).

Vectors vs. Scalars

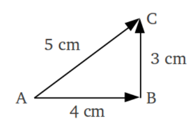
Example

Classify each quantity as a vector or a scalar.

1. The temperature outside is 3°C (scalar)
2. Gravity causes a ball to accelerate at 9.8 m/s^2 downward (vector)
3. A student has a mass of 68 kg (scalar)
4. A car drives 25 km west (vector)

Vector Notation

Vectors are often represented using arrows, since arrows have both a length (magnitude) and a direction.



The three vectors are $\vec{AB}$, $\vec{BC}$ and $\vec{AC}$.

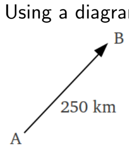
The magnitude of a vector is usually indicated using vertical bars (absolute value).

$$|\vec{AB}| = 4 \qquad |\vec{BC}| = 3 \qquad |\vec{AC}| = 5$$

Representing Vectors

Vectors can be represented in many ways.

1. Using a diagram



2. Using words (250 km northeast)
3. Using symbols ($\vec{AB}$)

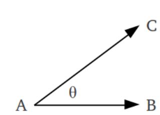
Terminology

Equivalent vectors are those with the same magnitude and direction (e.g. 9.8 m/s^2 down and 9.8 m/s^2 down).

Opposite vectors are those with the same magnitude, but have the opposite direction (e.g. 5 km NE and 5 km SW). The vector $-\vec{v}$ is opposite to $\vec{v}$.

Parallel vectors may have different magnitudes, but their directions are either the same or opposite (e.g. 3 N left, 15 N right).

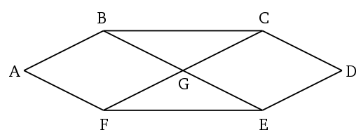
The *angle between two vectors*, θ, is the acute or obtuse angle formed between them when drawn tail-to-tail.



Terminology

Example

For $\vec{AB}$ below, state two equivalent vectors, two opposite vectors, and two parallel vectors.



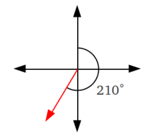
Equivalent: $\vec{FG}$, $\vec{GC}$ and $\vec{ED}$.

Opposite: $\vec{CG}$, $\vec{GF}$, $\vec{DE}$ and $\vec{BA}$.

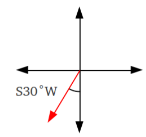
Parallel: all seven above, plus $\vec{FC}$ and $\vec{CF}$.

Bearings

True bearings rotate clockwise, beginning at the north.

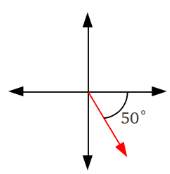


Quadrant bearings measure the angle east or west of the north-south line.



Bearings

Express the following vector using both a true bearing and quadrant bearing.



The true bearing is 140°, and the quadrant bearing is S40°E.

Unit Vectors

A *unit vector*, $\hat{v}$, is a vector with a magnitude of 1. That is, $|\hat{v}| = 1$.

Since $k \cdot \frac{1}{k} = 1$, a unit vector in the direction of any vector $\vec{v}$ can be found by multiplying $\vec{v}$ by the reciprocal of its magnitude.

$\hat{v} = \dfrac{1}{|\vec{v}|} \cdot \vec{v}$ or $\hat{v} = \dfrac{\vec{v}}{|\vec{v}|}$

Any vector, $\vec{v}$, can be expressed as the product of its magnitude, $|\vec{v}|$ and a unit vector, $\hat{v}$.

$\vec{v} = \hat{v} \cdot |\vec{v}|$

Unit Vectors

Example

Vector $\vec{v}$ has a magnitude of $\frac{5}{2}$ units, with a bearing of 35°. Determine two unit vectors parallel to $\vec{v}$.

Since $\frac{5}{2} \cdot \frac{2}{5} = 1$, vector $\vec{a} = \frac{2}{5}\vec{v}$ is a unit vector parallel to $\vec{v}$.

Since $-\vec{v}$ is parallel to $\vec{v}$, $\vec{b} = -\frac{2}{5}\vec{v}$ is another unit vector parallel to $\vec{v}$.

Questions?