MCV4U: Calculus & Vectors

Algebraic Vectors In Two- and Three-Space

Part 1: Representing Vectors

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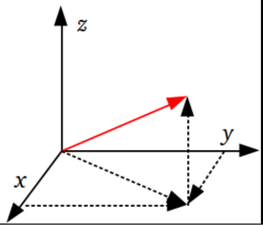


Unit Vectors in Two-Space

Recall that vectors can be "moved", as long as their directions and magnitudes are left unchanged.

A *position vector* (v_x, v_y) is a vector such that its tail is located at the origin, and its head is located at point $P(x, y)$.

Recall that any vector in two-space can be expressed as some *linear combination* of two non-collinear vectors.

Two such vectors are $\hat{\imath}$ and $\hat{\jmath}$, which are unit vectors in the direction of the positive x- and y-axes.

Using these two unit vectors, we can create any vector in the xy-plane.

Unit Vectors in Two-Space

Example

Express the position vector $\vec{v} = (2, -8)$ using the unit vectors $\hat{\imath}$ and $\hat{\jmath}$.

$$(2, -8) = 2\hat{\imath} - 8\hat{\jmath}$$

Example

Express the vector $10\hat{\imath}$ as a position vector.

$$10\hat{\imath} = (10, 0)$$

Drawing Vectors in Two-Space

To draw a vector in two-space, we can use the Cartesian Plane (x- and y-axes) and place the vector's tail at the origin.

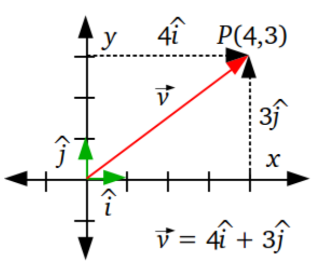
This is why these vectors are sometimes referred to as *Cartesian vectors*.

Cartesian vectors, as we will see, offer a way to perform calculations involving vectors using algebraic techniques, rather than geometric methods.

Drawing Vectors in Two-Space

Example

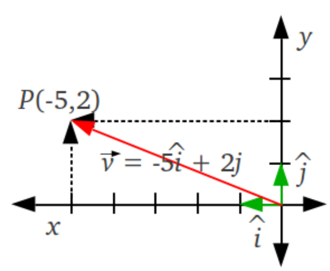
Draw a diagram for the position vector $\vec{v} = (4, 3)$.



Drawing Vectors in Two-Space

Example

Draw a diagram for the position vector $\vec{v} = (-5, 2)$.



Vectors in Three-Space

We can extend our knowledge of vectors beyond two-dimensions, from *two-space* into *three-space*.

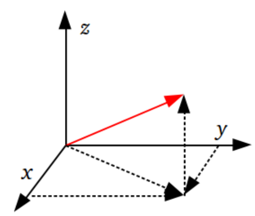
Like two-space, a position vector $\vec{v} = (v_x, v_y, v_z)$ in three-space is a vector that has been translated such that its tail is at the origin and its head at some point $P(x, y, z)$.

A position vector (v_x, v_y, v_z) describes any vector in three-space that has the same magnitude and direction.

Vectors in Three-Space

One thing we may need to do is to visualize vectors in three-space.

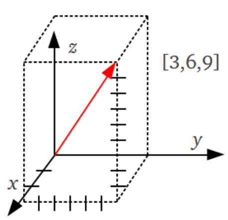
The x-axis is typically drawn coming out of the page, while the y and z-axes are what we typically associate with a two-dimensional grid.



Vectors in Three-Space

One way to visualize vectors is to use a box with edges relative to the x, y and z-axes.

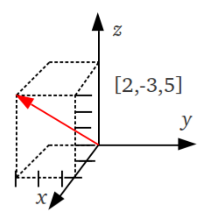
For example, the vector $(3, 6, 9)$ is drawn below.



Vectors in Three-Space

Example

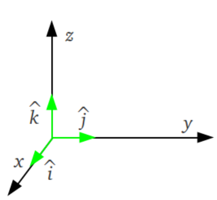
Draw a diagram for the vector $(2, -3, 5)$.



Unit Vectors in Three-Space

Similar to vectors in two-space, any vector in three-space can be represented as a linear combination of three non-coplanar, non-zero vectors.

The simplest example is to use the three unit vectors, $\hat{i}$, $\hat{j}$ and $\hat{k}$, located along the x, y and z-axes.



Unit Vectors in Three-Space

Linear Combinations of Vectors In Three-Space

For any position vector $\vec{v} = (v_x, v_y, v_z)$ in three-space, $\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$.

A proof can be constructed using components, or by using addition and subtraction of unit vectors.

This allows us to express position vectors using unit vectors, or vice versa.

Unit Vectors in Three-Space

Example

Express the position vector $\vec{v} = (6, -3, 5)$ using the three unit vectors $\hat{i}$, $\hat{j}$ and $\hat{k}$.

$$(6, -3, 5) = 6\hat{i} - 3\hat{j} + 5\hat{k}$$

Example

Express the vector $5\hat{i} - 2\hat{k}$ as a position vector.

$$5\hat{i} - 2\hat{k} = (5, 0, -2)$$

Questions?