MCV4U: Calculus & Vectors

Linear Systems Involving 3 Variables

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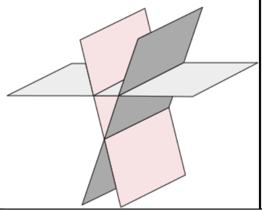

Linear Systems Involving 2 Variables

Example

Solve the system of equations below.

$$\begin{aligned} 5x + 3y &= 5 \\ 3x + 4y &= 14 \end{aligned}$$

Multiply the first equation by 4 and the second by 3 to eliminate y.

$$\begin{aligned} 20x + 12y &= 20 \\ -\quad 9x + 12y &= 42 \\ 11x &= -22 \\ x &= -2 \end{aligned}$$

When $x = -2$, $y = 5$, so the solution is $(-2, 5)$.

Linear Systems Involving 3 Variables

The same technique used to solve linear systems with 2 variables can be extended to 3 or more variables.

By eliminating one variable (say z) from one pair of equations, we can create a new equation with only two variables in it (x and y).

Using a *different* pair of equations, we can eliminate the same variable and create another new equation with the same two variables.

Using these two new equations, we can solve for the two unknown variables (x and y).

We can then substitute these values into one of the original equations to determine the final value (z).

Sometimes it may be possible to skip one or more steps.

Linear Systems Involving 3 Variables

Example

Solve the system of equations below.

$$\begin{aligned} 2x + y - z &= -3 \\ 4x - y + z &= 12 \\ y + z &= 2 \end{aligned}$$

Since the third equation does not involve x at all, create a new equation without x using the first and second equations (multiplying the first by 2).

$$\begin{aligned} 4x + 2y - 2z &= -6 \\ -\quad 4x - y + z &= 12 \\ 3y - 3z &= -18 \\ y - z &= -6 \end{aligned}$$

Linear Systems Involving 3 Variables

Use this new equation, and the third, to solve for y and z.

$$\begin{aligned} y + z &= 2 \\ +\quad y - z &= -6 \\ 2y &= -4 \\ y &= -2 \end{aligned}$$

Substitute $y = -2$ into one of the equations above.

$$\begin{aligned} -2 + z &= 2 \\ z &= 4 \end{aligned}$$

Use $y = -2$ and $z = 4$ in one of the original equations.

$$\begin{aligned} 2x + (-2) - 4 &= -3 \\ x &= \tfrac{3}{2} \end{aligned}$$

The solution is $\left(\frac{3}{2}, -2, 4\right)$.

Linear Systems Involving 3 Variables

Example

Solve the system of equations below.

$$\begin{aligned} x - 3y + 3z &= -4 \\ 2x + 3y - z &= 15 \\ 4x - 3y - z &= 19 \end{aligned}$$

y can be eliminated in two ways: using the first and second equations, and using the second and third equations.

$$\begin{aligned} x - 3y + 3z &= -4 \\ +\quad 2x + 3y - z &= 15 \\ 3x + 2z &= 11 \end{aligned} \qquad \begin{aligned} 2x + 3y - z &= 15 \\ +\quad 4x - 3y - z &= 19 \\ 6x - 2z &= 34 \end{aligned}$$

Linear Systems Involving 3 Variables

Use the two new equations to solve for x and z.

$$\begin{aligned} 3x + 2z &= 11 \\ + \quad 6x - 2z &= 34 \\ 9x &= 45 \\ x &= 5 \end{aligned}$$

Substitute $x = 5$ into one of the equations above.

$$\begin{aligned} 3(5) + 2z &= 11 \\ z &= -2 \end{aligned}$$

Use $x = 5$ and $z = -2$ in one of the original equations.

$$\begin{aligned} 2(5) + 3y - (-2) &= 15 \\ y &= 1 \end{aligned}$$

The solution is $(5, 1, -2)$.

Linear Systems Involving 3 Variables

Example

Solve the system of equations below.

$$\begin{aligned} x + y - z &= 4 \\ x - 2y + 3z &= -6 \\ 2x + 3y + z &= 7 \end{aligned}$$

z can be eliminated by adding the first and third equations, and by multiplying the first equation by 3 and adding it to the second equation.

$$\begin{aligned} x + y - z &= 4 \\ + \quad 2x + 3y + z &= 7 \\ 3x + 4y &= 11 \end{aligned} \qquad \begin{aligned} 3x + 3y - 3z &= 12 \\ + \quad x - 2y + 3z &= -6 \\ 4x + y &= 6 \end{aligned}$$

Linear Systems Involving 3 Variables

Solve for x and y using the two new equations (multiplying the second by 4 to eliminate y).

$$\begin{aligned} 3x + 4y &= 11 \\ - \quad 16x + 4y &= 24 \\ -13x &= -13 \\ x &= 1 \end{aligned}$$

Using the first of the two new equations, $3(1) + 4y = 11$ yields $y = 2$.

Using the first of the original equations, $1 + 2 - z = 4$ yields $z = -1$.

The solution is $(1, 2, -1)$.

Linear Systems Involving 3 Variables

Example

Solve the system of equations below.

$$\begin{aligned} x + 2y + 6z &= 5 \\ x - y + 2z &= -3 \\ x - 4y - 2z &= 1 \end{aligned}$$

Eliminate z by using the first and second equations, then the first and third.

$$\begin{aligned} x + 2y + 6z &= 5 \\ - \quad x - y + 2z &= -3 \\ 3x + 4y &= 8 \end{aligned} \qquad \begin{aligned} x + 2y + 6z &= 5 \\ - \quad x - 4y - 2z &= 1 \\ 6x + 8y &= 4 \end{aligned}$$

Linear Systems Involving 3 Variables

Solve for x and y by halving the second equation.

$$\begin{aligned} 3x + 4y &= 8 \\ - \quad 3x + 4y &= 2 \\ 0 &= 6 \end{aligned}$$

Since this equation cannot be true, the system is inconsistent. There are no real solutions.

Questions?