MCV4U: Calculus & Vectors

Intersections of Lines

Part 1: Lines in R^2

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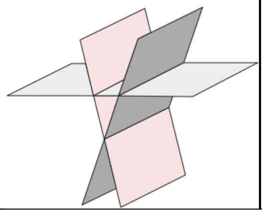

Intersections of Lines In Two-Space

When graphed, two lines in two-space may intersect or they may not.

There are three possibilities in two-space.

- The lines are *parallel* (no point of intersection)
- The lines are *coincident* (infinite points of intersection)
- The lines intersect at a single point

To find the point(s) of intersection, we use a system of linear equations.

A linear system that has one or infinitely many points of intersection is *consistent*, while a system with no solutions is *inconsistent*.

Intersections of Lines In Two-Space

Example

Determine whether the linear system below is consistent.

$$L_1 : x = 2 - 2t \qquad L_2 : x = 5 + 6s$$
$$y = 1 + 3t \qquad y = 3 - 9s$$

Direction vectors for the two lines are $\vec{m_1} = (-2, 3)$ and $\vec{m_2} = (6, -9)$.

Since $\vec{m_2} = -3\vec{m_1}$, the lines are either parallel or coincident.

Intersections of Lines In Two-Space

Substitute the point $(2, 1)$, which is on L_1, into the parametric equations for L_2.

$$2 = 5 + 6s \qquad 1 = 3 - 9s$$
$$s = -\frac{1}{2} \qquad s = \frac{2}{9}$$

Since the values of the parameter s are different, the lines are parallel (inconsistent).

This is usually a good check to perform before attempting to solve a linear system.

Intersections of Lines In Two-Space

Example

Determine the point of intersection of the lines $3x - 8y + 5 = 0$ and $2x + 4y + 1 = 0$.

Multiply the second equation by 2, then add the new equation to the first to eliminate the y variable.

$$\begin{aligned} & 3x - 8y + 5 = 0 \\ + \quad & 4x + 8y + 2 = 0 \\ & 7x + 7 = 0 \\ & x = -1 \end{aligned}$$

When $x = -1$, $y = \frac{1}{4}$, so the point of intersection is $\left(-1, \frac{1}{4}\right)$.

Intersections of Lines In Two-Space

Example

Determine the point of intersection of the lines $\vec{r_1} = (1, 3) + t(-5, 2)$ and $\vec{r_2} = (2, 4) + s(1, -3)$.

Convert each line to its parametric equations.

$$x = 1 - 5t \qquad x = 2 + s$$
$$y = 3 + 2t \qquad y = 4 - 3s$$

Intersections of Lines In Two-Space

Equate the x and y values to create a system of equations with variables s and t.

$$\begin{aligned} 1 - 5t &= 2 + s \\ 3 + 2t &= 4 - 3s \end{aligned}$$

Multiply the first equation by 3 and add it to the second equation.

$$\begin{aligned} 3 - 15t &= 6 + 3s \\ + \quad 3 + 2t &= 4 - 3s \\ 6 - 13t &= 10 \\ t &= -\tfrac{4}{13} \end{aligned}$$

Intersections of Lines In Two-Space

Substitute $t = -\frac{4}{13}$ in the parametric equations for $\vec{L_1}$.

$$\begin{aligned} x &= 1 - 5\left(-\tfrac{4}{13}\right) \qquad & y &= 3 + 2\left(-\tfrac{4}{13}\right) \\ &= \tfrac{33}{13} & &= \tfrac{31}{13} \end{aligned}$$

The point of intersection occurs at $\left(\frac{33}{13}, \frac{31}{13}\right)$.

Questions?