MCV4U: Calculus & Vectors

Forces as Vectors

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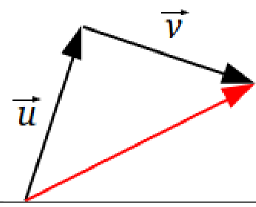


Forces

A *force* is a push or a pull on an object.

A force has both a magnitude and a direction, so it can be represented by a vector.

A force, $\vec{F}$, can be calculated using the relationship $\vec{F} = m\vec{a}$, where m is the object's mass and $\vec{a}$ is its acceleration.

In many cases we use the acceleration due to gravity, which is approximately 9.8 m/s^2.

Forces have Newtons (N) as units, or kg·m/s^2.

Multiple Forces Acting On an Object

Example

Two forces of 8 N and 15 N act at right angles to each other. Determine the magnitude and direction of the resultant.

Use the Pythagorean Theorem to calculate the magnitude of the resultant force.

$$|\vec{r}| = \sqrt{8^2 + 15^2}$$
$$= 17 \text{ N}$$

Multiple Forces Acting On an Object

Use a trigonometric ratio to determine the angle between the two forces.

$$\theta = \tan^{-1}\left(\frac{8}{15}\right)$$
$$\approx 28^\circ$$

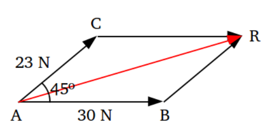
The resultant force has a magnitude of 17 N, at an angle of approximately 28° relative to the 15 N force.

Multiple Forces Acting On an Object

Example

Two children pull a sled, one with a force of 30 N [E] and the other with a force of 23 N [NE]. Determine the magnitude, and direction, of the resultant force.

Use the following diagram, where $\vec{AB}$ is 30 N force, $\vec{AC}$ is the 23 N force, and $\vec{AR}$ is the resultant force.



Multiple Forces Acting On an Object

We need to determine the magnitude of $\vec{AR}$.

From the given information, $\angle CAB = 45^\circ$, so $\angle ABR = 180^\circ - 45^\circ = 135^\circ$.

Use the cosine law to determine $|\vec{AR}|$.

$$|\vec{AR}| = \sqrt{|\vec{AB}|^2 + |\vec{AC}|^2 - 2(|\vec{AB}|)(|\vec{AC}|)\cos(ABR)}$$
$$= \sqrt{30^2 + 23^2 - 2(30)(23)\cos(135^\circ)}$$
$$\approx 49 \text{ N}$$

Multiple Forces Acting On an Object

To determine the direction, use the sine law (or cosine law) to find the measure of $\angle CAR$, then add 45° (since $\vec{AC}$ faces northeast).

$$\frac{\sin(CAR)}{|\vec{BC}|} = \frac{\sin(ABR)}{|\vec{AR}|}$$
$$\frac{\sin(CAR)}{30} \approx \frac{\sin(135°)}{49}$$
$$\angle CAR \approx \sin^{-1}\left(\frac{30\sin(135°)}{49}\right)$$
$$\approx 26°$$

Therefore, the resultant force has a bearing of approximately $26° + 45°$, or 71°. Its magnitude is 49 N.

Equilibrium

An object is in *equilibrium* if forces act on it, but it does not move.

Thus, for any object in equilibrium, the sum of the forces acting on it is the zero vector.

A force that counterbalances the resultant, keeping the object in equilibrium, is called an *equilibrant*.

The equilibrant is equal in magnitude to the resultant, but has opposite direction.

Tension

Tension is a pulling force, directed away from an object that is in equilibrium.

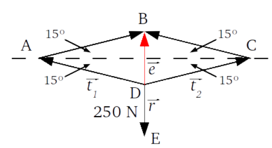
When solving problems involving tension, we can use information about the resultant and equilibrant forces.

Tension

Example

A 250 N weight is suspended by two ropes, each making angles of 15° below the horizontal. Determine the tensions in the ropes.

Use the following diagram, where $\vec{t_1}$ and $\vec{t_2}$ represent the tensions in the ropes, and $\vec{r}$ and $\vec{e}$ are the resultant and equilibrant respectively, each with a force of 250 N.



Tension

$\angle ABD = (180° - 30°) \div 2 = 75°$.

Use the sine law to determine the magnitude of $\vec{t_1}$.

$$\frac{|\vec{t_1}|}{\sin(ABD)} = \frac{|\vec{r}|}{\sin(DAB)}$$
$$\frac{|\vec{t_1}|}{\sin(75°)} = \frac{250}{\sin(30°)}$$
$$|\vec{t_1}| = \frac{250\sin(75°)}{\sin(30°)}$$
$$\approx 483 \text{ N}$$

Since $|\vec{t_1}| = |\vec{t_2}|$, the tension in each rope is approximately 483 N.

Tension

Example

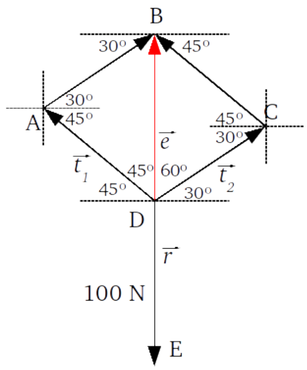
A sign with a mass of 10.2 kg hangs from two wires. One wire makes an angle of 45° with the horizontal, the other an angle of 30°. Determine the tension in each wire.

First, determine the force of the equilibrant acting downward on the sign.

$$|\vec{e}| \approx 10.2 \times 9.8$$
$$\approx 100 \text{ N}$$

Use the following diagram, where $\vec{t_1}$ and $\vec{t_2}$ represent the tensions in the wires, and $\vec{r}$ and $\vec{e}$ are the resultant and equilibrant respectively, each with a force of 100 N.

Applications of Vectors



Applications of Vectors

$\angle ABD = 90° - 30° = 60°$.

Use the sine law to determine the magnitude of $\vec{t_1}$.

$$\frac{|\vec{t_1}|}{\sin(ABD)} = \frac{|\vec{r}|}{\sin(DAB)}$$
$$\frac{|\vec{t_1}|}{\sin(60°)} \approx \frac{100}{\sin(75°)}$$
$$|\vec{t_1}| \approx \frac{100\sin(60°)}{\sin(75°)}$$
$$\approx 90 \text{ N}$$

The tension in the wire at 45° is approximately 90 N.

Applications of Vectors

$\angle DBC = 90° - 45° = 45°$.

Use the sine law to determine the magnitude of $\vec{t_2}$.

$$\frac{|\vec{t_2}|}{\sin(DBC)} = \frac{|\vec{r}|}{\sin(DCB)}$$
$$\frac{|\vec{t_2}|}{\sin(45°)} \approx \frac{100}{\sin(75°)}$$
$$|\vec{t_2}| \approx \frac{100\sin(45°)}{\sin(75°)}$$
$$\approx 73 \text{ N}$$

The tension in the wire at 30° is approximately 73 N.

Questions?