MCV4U: Calculus & Vectors

Derivatives of Other Trigonometric Functions
(Tangent, Secant, Cosecant and Cotangent)

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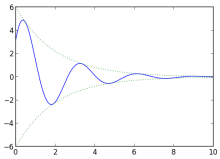

Derivatives Involving Sinusoidal Functions

Recap

Determine the derivative of $f(x) = \dfrac{1-\sin x}{1+\sin x}$.

$$\begin{aligned} f'(x) &= \frac{-\cos x(1+\sin x) - \cos x(1-\sin x)}{(1+\sin x)^2} \\ &= \frac{-\cos x - \sin x\cos x - \cos x + \sin x\cos x}{(1+\sin x)^2} \\ &= -\frac{2\cos x}{(1+\sin x)^2} \end{aligned}$$

Derivative of $f(x) = \tan x$

The derivatives of other trigonometric functions, can be found by expressing them in terms of $\sin x$ and $\cos x$.

Since $f(x) = \tan x = \dfrac{\sin x}{\cos x}$, use the quotient rule.

$$\begin{aligned} f'(x) &= \frac{\cos x\cos x - \sin x(-\sin x)}{\cos^2 x} \\ &= \frac{\sin^2 x + \cos^2 x}{\cos^2 x} \\ &= \frac{1}{\cos^2 x} \\ &= \sec^2 x \end{aligned}$$

Derivative of the Tangent Function

If $f(x) = \tan x$, then $f'(x) = \sec^2 x$.

Derivatives Involving $\tan x$

Example

Determine the derivative of $f(x) = 4\tan^3 x - \tan x$.

$$\begin{aligned} f'(x) &= 12\tan^2 x\sec^2 x - \sec^2 x \\ &= (12\tan^2 x - 1)\sec^2 x \end{aligned}$$

Example

Determine the derivative of $y = x\tan 3x^2$.

$$\begin{aligned} \frac{dy}{dx} &= 1\cdot\tan 3x^2 + x\cdot\sec^2 3x^2\cdot 6x \\ &= \tan 3x^2 + 6x^2\sec^2 3x^2 \end{aligned}$$

Derivatives Involving $\tan x$

Example

Determine the derivative of $f(x) = \dfrac{1}{\tan x}$.

$$\begin{aligned} f'(x) &= \frac{0(\tan x) - 1(\sec^2 x)}{\tan^2 x} \\ &= -\frac{sec^2 x}{\tan^2 x} \\ &= -\frac{1}{\cos^2 x}\cdot\frac{\cos^2 x}{\sin^2 x} \\ &= -\frac{1}{\sin^2 x} \\ &= -\csc^2 x \end{aligned}$$

Derivatives of Secondary Trig. Functions

Since $\dfrac{1}{\tan x} = \cot x$, the previous example gives us the derivative for $f(x) = \cot x$.

Derivatives of Secondary Trigonometric Functions

The secondary (reciprocal) trigonometric ratios are $\cot x$, $\sec x$ and $\csc x$. Their derivaties are:

- $\frac{d}{dx}(\cot x) = -\csc^2 x$
- $\frac{d}{dx}(\sec x) = \sec x\tan x$
- $\frac{d}{dx}(\csc x) = -\csc x\cot x$

The derivatives of secant and cosecant are relatively easy to prove, and are left a homework exercise.

Derivatives of Secondary Trig. Functions

Example

Determine $\left.\frac{dy}{dx}\right|_{x=\frac{\pi}{4}}$ if $y = \csc^2 x$.

$$\begin{aligned}
\frac{dy}{dx} &= 2\csc x(-\csc x \cot x) \\
&= -2\csc^2 x \cot x \\
\left.\frac{dy}{dx}\right|_{x=\frac{\pi}{4}} &= -2\csc^2\left(\tfrac{\pi}{4}\right)\cot\left(\tfrac{\pi}{4}\right) \\
&= -2(2)(1) \\
&= -4
\end{aligned}$$

Derivatives of Secondary Trig. Functions

Example

Determine $\frac{dy}{dx}$ if $y = \dfrac{x}{\cot x}$.

$$\begin{aligned}
\frac{dy}{dx} &= \frac{\cot x - x(-\csc^2 x)}{\cot^2 x} \\
&= \frac{\cot x}{\cot^2 x} + \frac{x\csc^2 x}{\cot^2 x} \\
&= \frac{1}{\cot x} + x \cdot \frac{1}{\sin^2 x} \cdot \frac{\sin^2 x}{\cos^2 x} \\
&= \tan x + x\sec^2 x
\end{aligned}$$

Questions?