MCV4U: Calculus & Vectors

# Absolute Extrema

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## Local Extrema

**Recap**

Determine the nature of any local extrema for the function $f(x) = 2x^3 + 6x^2 - 18x$.

Since $f(x)$ has degree 3, there can be at most 2 local extrema. Find the first derivative to identify critical points.

$$\begin{aligned} f'(x) &= 6x^2 + 12x - 18 \\ &= 6(x+3)(x-1) \end{aligned}$$

Critical points are at $x = -3$ and $x = 1$. Since $f(-3) = 54$ and $f(1) = -10$, there are points at $(-3, 54)$ and $(1, -10)$.





## Local Extrema

Find the second derivative.

$$\begin{aligned} f''(x) &= 12x + 12 \\ &= 12(x+1) \end{aligned}$$

Evaluate $f''(x)$ at the critical values.

| $x$ | $-3$ | $1$ |
|---|---|---|
| $f''(x)$ | $-24$ | $24$ |
| sign | $-$ | $+$ |

Since $f''(-3) < 0$, there is a local maximum at $(-3, 54)$.

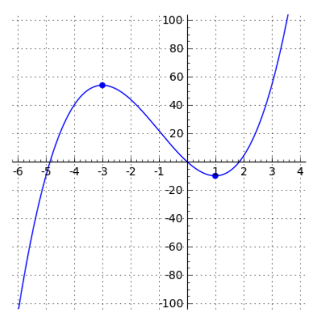
Since $f''(1) > 0$, there is a local minimum at $(1, -10)$.





## Local Extrema

A graph of $f(x)$ is below.








## Absolute Extrema

A local extremum may be an *absolute extremum* (or *global extremum*) if it is the maximum or minimum point that a specified function reaches across its entire domain.

Some functions, such as even-degree polynomials or sinusoidal functions, have absolute extrema.

Other functions, such as odd-degree polynomials or logarithmic functions, do not have absolute extrema, as their ranges are $(-\infty, \infty)$.

In some cases, a function have its domain restricted. If so, an absolute extremum may occur in one of three places.

**Absolute Extrema On a Specified Interval**

For a function $f(x)$ on domain $[a, b]$, absolute extrema may occur at local extrema, when $x = a$, or when $x = b$.





## Absolute Extrema

**Example**

Determine any absolute extrema for $f(x) = 3x^4 - 20x^3 + 12x^2 + 96x$.

Since $f(x)$ is a polynomial with order 4 and a positive leading coefficient, there should be an absolute minimum somewhere.

Find the first derivative to identify critical points.

$$\begin{aligned} f'(x) &= 12x^3 - 60x^2 + 24x + 96 \\ &= 12(x^3 - 5x^2 + 2x + 8) \end{aligned}$$

## Absolute Extrema

Use polynomial division to factor the cubic expression.

| | | | | |
|---|---|---|---|---|
| | 1 | −5 | 2 | 8 |
| 2 | | 2 | −6 | −8 |
| | 1 | −3 | −4 | 0 |

$$\begin{aligned} f'(x) &= 12(x-2)(x^2-3x-4) \\ &= 12(x-2)(x-4)(x+1) \end{aligned}$$

Critical points occur at $x = 2$, $x = 4$ and $x = -1$.





## Absolute Extrema

Find the second derivative.

$$\begin{aligned} f''(x) &= 36x^2 - 120x + 24 \\ &= 12(3x^2 - 10x + 2) \end{aligned}$$

Evaluate $f''(x)$ at the critical values.

| $x$ | $-1$ | $2$ | $4$ |
|---|---|---|---|
| $f''(x)$ | 180 | $-72$ | 120 |
| sign | $+$ | $-$ | $+$ |

Since both $f''(-1)$ and $f''(4)$ are positive, one of these critical points is an absolute minimum.

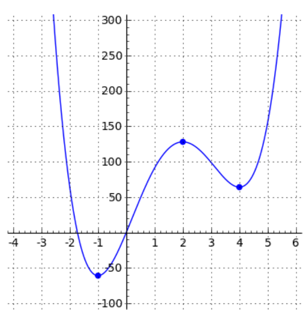
Since $f(-1) = -61 < f(4) = 64$, there is an absolute minimum at $(-1, -61)$.





## Absolute Extrema

A graph of $f(x)$ is below.








## Absolute Extrema

**Example**

Determine any absolute extrema for $y = x^3 - 9x^2 + 15x$ on the interval $[0, 8]$.

Since $y|_{x=0} = 0$ and $y|_{x=8} = 56$, a possible absolute minimum is at $(0, 0)$ while a possible absolute maximum is at $(8, 56)$.

Find the first derivative to identify critical points.

$$\begin{aligned} \frac{dy}{dx} &= 3x^2 - 18x + 15 \\ &= 3(x-1)(x-5) \end{aligned}$$

Therefore, critical points are $x = 1$ and $x = 5$.





## Absolute Extrema

Find the second derivative.

$$\begin{aligned} \frac{d^2y}{dx^2} &= 6x - 18 \\ &= 6(x-3) \end{aligned}$$

Test critical points to confirm they are local extrema.

| $x$ | 1 | 5 |
|---|---|---|
| $f''(x)$ | $-12$ | 12 |
| sign | $-$ | $+$ |

There is a local maximum at $x = 1$ and a local minimum at $x = 5$.





## Absolute Extrema

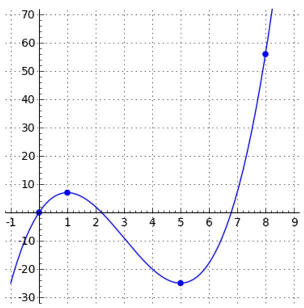
Since $y|_{x=1} = 7$ and $y|_{x=5} = -25$, there are local extrema at $(1, 7)$ and $(5, -25)$.

The absolute minimum of the function is at $(5, -25)$, which is a local minimum.

The absolute maximum of the function is at $(8, 56)$, which is at one end of the interval.

## Absolute Extrema

A graph of $y$ is below.








## Questions?