MCR3U: Functions

Transformations of Functions

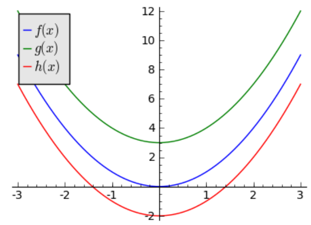
Part 1: Horizontal and Vertical Translations

J. Garvin

Vertical Translations

Investigate (Part 1)

On the same grid, graph the functions $f(x) = x^2$, $g(x) = x^2 + 3$ and $h(x) = x^2 - 2$.



Vertical Translations

A *vertical translation* is a transformation that shifts the graph of a function up or down.

Vertical translations do not change the shape of the graph, just its position.

Vertical translations have the form $g(x) = f(x) + k$.

- If $k > 0$, the graph shifts up k units.
- If $k < 0$, the graph shifts down k units.

The range of the transformed function can be determined by translating all discontinuities (holes, asymptotes) by k units.

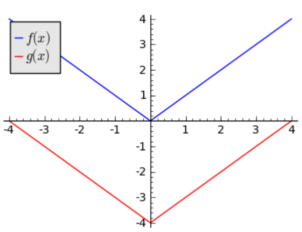
Vertical Translations

Example

Graph the functions $f(x) = |x|$ and $g(x) = |x| - 4$, and state the domain and range of $g(x)$.

x	Point	Image
-2	$(-2, 2)$	$(-2, -2)$
-1	$(-1, 1)$	$(-1, -3)$
0	$(0, 0)$	$(0, -4)$
1	$(1, 1)$	$(1, -3)$
2	$(2, 2)$	$(2, -2)$

Vertical Translations



Domain: $\{x \in R\}$

Range: $\{g(x) \in R \mid g(x) \geq -4\}$

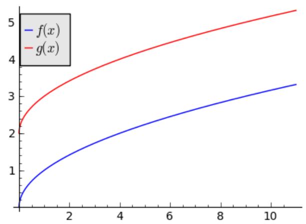
Vertical Translations

Example

Graph the functions $f(x) = \sqrt{x}$ and $g(x) = \sqrt{x} + 2$, and state the domain and range of $g(x)$.

x	Point	Image
0	$(0, 0)$	$(0, 2)$
1	$(1, 1)$	$(1, 3)$
4	$(4, 2)$	$(4, 4)$
9	$(9, 3)$	$(9, 5)$

Vertical Translations



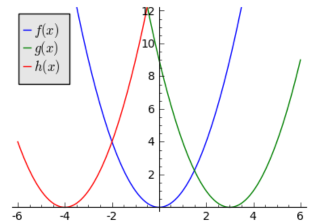
Domain: $\{x \in R \mid x \geq 0\}$

Range: $\{g(x) \in R \mid g(x) \geq 2\}$

Horizontal Translations

Investigate (Part 2)

On the same grid, graph the functions $f(x) = x^2$, $g(x) = (x-3)^2$ and $h(x) = (x+4)^2$.



Horizontal Translations

A *horizontal translation* is a transformation that shifts the graph of a function left or right.

Like vertical translations, horizontal translations change only a graph's position.

Horizontal translations have the form $g(x) = f(x-h)$.

- If $h > 0$, the graph shifts right h units.
- If $h < 0$, the graph shifts left h units.

The domain of the transformed function can be determined by translating all discontinuities (holes, asymptotes) by h units.

Horizontal Translations

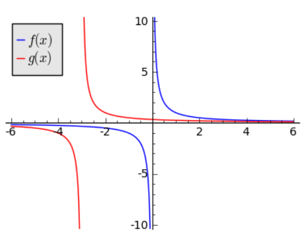
Example

Graph the functions $f(x) = \frac{1}{x}$ and $g(x) = \frac{1}{x+3}$, and state the domain and range of $g(x)$.

The vertical asymptote shifts 3 units left, to $x = -3$. The horizontal asymptote remains unchanged at $y = 0$.

x	Point	Image
-1	$(-1,-1)$	$(-4,-1)$
1	$(1,1)$	$(-2,1)$

Horizontal Translations



Domain: $\{x \in R \mid x \neq -3\}$

Range: $\{g(x) \in R \mid g(x) \neq 0\}$

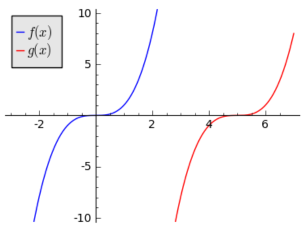
Horizontal Translations

Example

Graph the functions $f(x) = x^3$ and $g(x) = (x-5)^3$, and state the domain and range of $g(x)$.

x	Point	Image
-2	$(-2,-8)$	$(3,-8)$
-1	$(-1,-1)$	$(4,-1)$
0	$(0,0)$	$(5,0)$
1	$(1,1)$	$(6,1)$
2	$(2,8)$	$(7,8)$

Horizontal Translations



Domain: $\{x \in R\}$

Range: $\{g(x) \in R\}$

Vertical and Horizontal Translations

Example

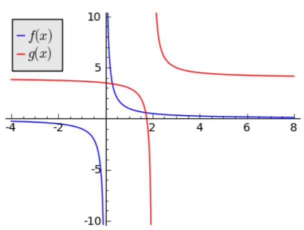
For the function $g(x) = \dfrac{1}{x-2} + 4$:

- State the parent function.
- Describe the transformations.
- Sketch a graph.
- Determine the domain and range.

The parent function is $f(x) = \frac{1}{x}$.

There is a vertical translation 4 units up, and a horizontal translation 2 units right.

Vertical and Horizontal Translations



Domain: $\{x \in R \mid x \neq 2\}$

Range: $\{g(x) \in R \mid g(x) \neq 4\}$

Questions?