

TRANSFORMATIONS

MCR3U: Functions

## Transformations of Functions

### Part 1: Horizontal and Vertical Translations

J. Garvin



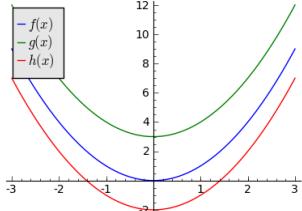
Slide 1/16

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## Vertical Translations

**Investigate (Part 1)**

On the same grid, graph the functions  $f(x) = x^2$ ,  $g(x) = x^2 + 3$  and  $h(x) = x^2 - 2$ .



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Slide 2/16

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## Vertical Translations

A *vertical translation* is a transformation that shifts the graph of a function up or down.

Vertical translations do not change the shape of the graph, just its position.

Vertical translations have the form  $g(x) = f(x) + k$ .

- If  $k > 0$ , the graph shifts up  $k$  units.
- If  $k < 0$ , the graph shifts down  $k$  units.

The range of the transformed function can be determined by translating all discontinuities (holes, asymptotes) by  $k$  units.

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Slide 3/16

TRANSFORMATIONS

## Vertical Translations

**Example**

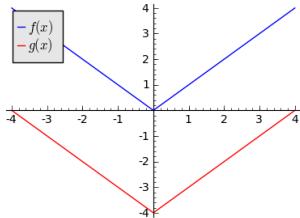
Graph the functions  $f(x) = |x|$  and  $g(x) = |x| - 4$ , and state the domain and range of  $g(x)$ .

$x$	Point	Image
-2	(-2, 2)	(-2, -2)
-1	(-1, 1)	(-1, -3)
0	(0, 0)	(0, -4)
1	(1, 1)	(1, -3)
2	(2, 2)	(2, -2)

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## Vertical Translations



Domain:  $\{x \in R\}$   
Range:  $\{g(x) \in R \mid g(x) \geq -4\}$

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Slide 5/16

TRANSFORMATIONS

## Vertical Translations

**Example**

Graph the functions  $f(x) = \sqrt{x}$  and  $g(x) = \sqrt{x} + 2$ , and state the domain and range of  $g(x)$ .

$x$	Point	Image
0	(0, 0)	(0, 2)
1	(1, 1)	(1, 3)
4	(4, 2)	(4, 4)
9	(9, 3)	(9, 5)

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Slide 6/16

TRANSFORMATIONS

TRANSFORMATIONS

Vertical Translations

The graph shows two parabolas on a Cartesian coordinate system. The blue parabola, labeled  $f(x)$ , passes through the origin  $(0,0)$ . The red parabola, labeled  $g(x)$ , also passes through the origin and is shifted vertically upwards by 2 units relative to  $f(x)$ .

Domain:  $\{x \in R \mid x \geq 0\}$   
 Range:  $\{g(x) \in R \mid g(x) \geq 2\}$

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 Slide 7/16

Horizontal Translations

Investigate (Part 2)

On the same grid, graph the functions  $f(x) = x^2$ ,  $g(x) = (x - 3)^2$  and  $h(x) = (x + 4)^2$ .

The graph shows three parabolas on a Cartesian coordinate system. The blue parabola, labeled  $f(x)$ , passes through the origin  $(0,0)$ . The green parabola, labeled  $g(x)$ , has its vertex at  $(3,0)$ . The red parabola, labeled  $h(x)$ , has its vertex at  $(-4,0)$ . All three parabolas have the same shape and orientation as  $f(x)$ .

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 Slide 8/16

TRANSFORMATIONS

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Horizontal Translations

A *horizontal translation* is a transformation that shifts the graph of a function left or right.

Like vertical translations, horizontal translations change only a graph's position.

Horizontal translations have the form  $g(x) = f(x - h)$ .

- If  $h > 0$ , the graph shifts right  $h$  units.
- If  $h < 0$ , the graph shifts left  $h$  units.

The domain of the transformed function can be determined by translating all discontinuities (holes, asymptotes) by  $h$  units.

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 Slide 9/16

Horizontal Translations

Example

Graph the functions  $f(x) = \frac{1}{x}$  and  $g(x) = \frac{1}{x+3}$ , and state the domain and range of  $g(x)$ .

The vertical asymptote shifts 3 units left, to  $x = -3$ . The horizontal asymptote remains unchanged at  $y = 0$ .

$x$	Point	Image
-1	(-1, -1)	(-4, -1)
1	(1, 1)	(-2, 1)

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 Slide 10/16

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TRANSFORMATIONS

Horizontal Translations

The graph shows two cubic functions on a Cartesian coordinate system. The blue curve, labeled  $f(x)$ , passes through the origin  $(0,0)$ . The red curve, labeled  $g(x)$ , has its local maximum at  $(5,0)$ . Both curves pass through the point  $(-3, -24)$ .

Domain:  $\{x \in R \mid x \neq -3\}$   
 Range:  $\{g(x) \in R \mid g(x) \neq 0\}$

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 Slide 11/16

Horizontal Translations

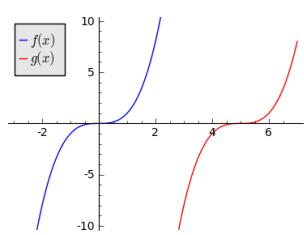
Example

Graph the functions  $f(x) = x^3$  and  $g(x) = (x - 5)^3$ , and state the domain and range of  $g(x)$ .

$x$	Point	Image
-2	(-2, -8)	(3, -8)
-1	(-1, -1)	(4, -1)
0	(0, 0)	(5, 0)
1	(1, 1)	(6, 1)
2	(2, 8)	(7, 8)

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 Slide 12/16

## Horizontal Translations



Domain:  $\{x \in R\}$

Range:  $\{g(x) \in R\}$

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Slide 13/16

## Vertical and Horizontal Translations

### Example

For the function  $g(x) = \frac{1}{x-2} + 4$ :

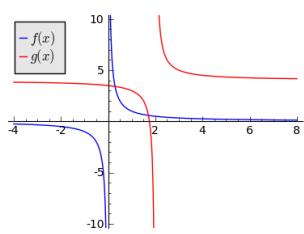
- State the parent function.
- Describe the transformations.
- Sketch a graph.
- Determine the domain and range.

The parent function is  $f(x) = \frac{1}{x}$ .

There is a vertical translation 4 units up, and a horizontal translation 2 units right.

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Slide 14/16

## Vertical and Horizontal Translations



Domain:  $\{x \in R \mid x \neq 2\}$

Range:  $\{g(x) \in R \mid g(x) \neq 4\}$

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Slide 15/16

## Questions?



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Slide 16/16