MCR3U: Functions

Transformations of Trigonometric Functions

Part 2: Multiple Transformations

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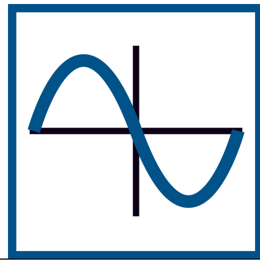


Multiple Transformations

Recall the key transformations/features of trigonometric functions of the form $y = af(b(x - c)) + d$.

- a is a vertical stretch, compression or reflection, and $|a|$ is the amplitude (for $\sin x$ and $\cos x$).
- b is a horizontal stretch, compression or reflection, and is *related to* the period.
- c is a horizontal translation, called a *phase shift*.
- d is a vertical translation, and gives the location of the *axis* (a horizontal line equidistant from the minimum and maximum points).

Multiple Transformations

Example

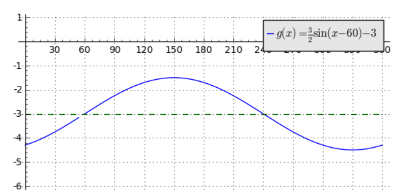
Identify the transformations applied to $f(x) = \sin x$ to produce $g(x) = \frac{3}{2}\sin(x - 60^\circ) - 3$, and describe key features of the graph of $g(x)$.

There is a vertical stretch by a factor of $\frac{3}{2}$. The amplitude of the function is $\frac{3}{2}$.

The graph has been translated 60° to the right. This is a phase shift of 60° to the right.

There is a vertical translation down by 3 units. The axis of the function is at $y = -3$.

Multiple Transformations

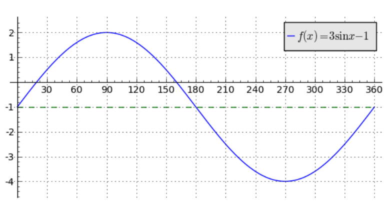


Multiple Transformations

Example

Sketch a graph of $f(x) = 3\sin x - 1$.

The axis is at $y = -1$, and the amplitude is 3. There is no phase shift, and the period remains 360°.



Multiple Transformations

Example

Sketch a graph of $f(x) = -4\cos(2x - 60^\circ) + 1$.

Remember that $f(x)$ must be written in fully-factored form, so $f(x) = -4\cos(2(x - 30^\circ)) + 1$.

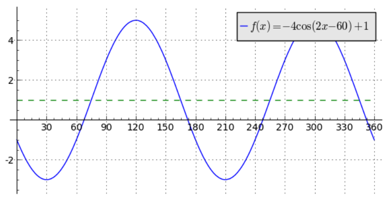
The amplitude is 4, and the axis is at $y = 1$, so the minimum and maximm values are -3 and 5 respectively.

Since there is a horizontal compression by a factor of 2, the period is $\frac{360}{2} = 180^\circ$.

There is a phase shift of 30° to the right. Since there is a vertical reflection, the cosine function will start at its minimum value, when $x = 30^\circ$.

Multiple Transformations

A graph of the transformed function is below.



Multiple Transformations

Example

Sketch a graph of $f(x) = -\frac{1}{2}\tan(x - 45^\circ) + 2$.

Just like the graphs of $f(x) = \sin x$ and $f(x) = \cos x$, the same transformations can be applied to $f(x) = \tan x$.

There is a vertical compression by a factor of $\frac{1}{2}$, so all points move half the distance toward the x-axis.

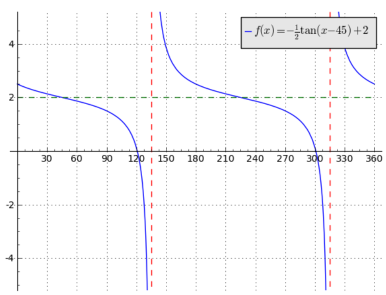
The vertical reflection moves all points above the x-axis below it, and vice versa, "flipping" the graph.

There is a phase shift of 45° to the right, and the axis is at $y = 2$.

Thus, a point like $(0, 0)$ which is on $f(x) = \tan x$ would have new coordinates $\left(0 + 45, -\frac{1}{2}(0) + 2\right) = (45, 2)$.

Multiple Transformations

A graph of the transformed function is below.



Questions?