MCR3U: Functions

Transformations of Trigonometric Functions

Part 1: Simple Transformations

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Transformations

Recall that a function, $f(x)$, may be transformed to a related function, $g(x)$ with the form $g(x) = af(b(x-c)) + d$.

- a is a vertical stretch/compression, and possibly a vertical reflection (in the x-axis).
- b is a horizontal stretch/compression, and possibly a horizontal reflection (in the y-axis).
- c is a horizontal translation.
- d is a vertical translation.

Transformations

If $f(x)$ is a trigonometric function, the same transformations apply, and correspond to certain features of the graph.

- a is a vertical stretch, compression or reflection, and $|a|$ is the amplitude (for $\sin x$ and $\cos x$).
- b is a horizontal stretch, compression or reflection, and is *related to* the period.
- c is a horizontal translation, called a *phase shift*.
- d is a vertical translation, and gives the location of the *axis* (a horizontal line equidistant from the minimum and maximum points).

Vertical Stretches/Compressions

Example

Identify the transformations applied to $f(x) = \sin x$ to produce $g(x) = 5 \sin x$, and describe how the attributes of $g(x)$ have changed.

There is a vertical stretch by a factor of 5. The amplitude of the function is 5 units.

Example

Identify the transformations applied to $f(x) = \cos x$ to produce $g(x) = \cos(x + 45^{c}irc)$, and describe how the attributes of $g(x)$ have changed.

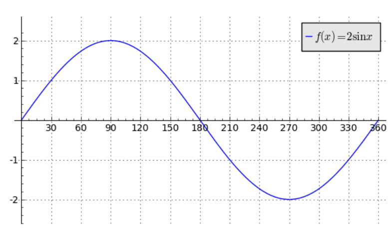
There is a horizontal translation 45° to the left. There is a phase shift of $-45°$, or a phase shift of 45° to the left.

Vertical Stretches/Compressions

Example

Sketch a graph of $f(x) = 2 \sin x$.

There is a vertical stretch by a factor of 2, so the amplitude is 2. All other aspects of $f(x) = \sin x$ are the same.

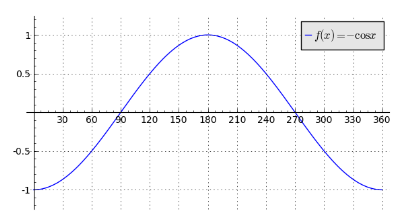


Vertical Reflections

Example

Sketch a graph of $f(x) = -\cos x$.

There is a vertical reflection, so instead of an $f(x)$-intercept at 1 like $f(x) = \cos x$, the $f(x)$-intecept is at -1 instead.

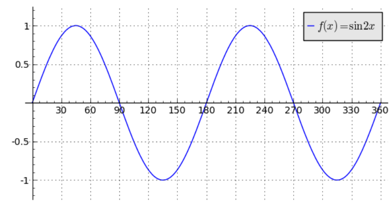


Horizontal Stretches/Compressions

Example

Sketch a graph of $f(x) = \sin 2x$.

There is a horizontal compression by a factor of $\frac{1}{2}$, so the period changes from $360°$ to $\frac{360}{2} = 180°$.

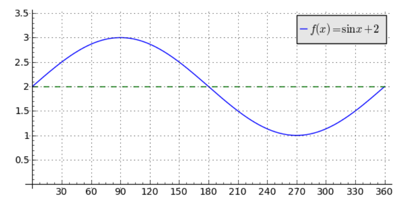


Vertical Translations

Example

Sketch a graph of $f(x) = \sin x + 2$.

The vertical translation has the effect of moving all points, and the axis, two units upward.

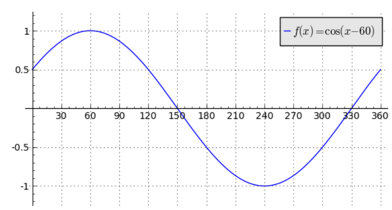


Horizontal Translations

Example

Sketch a graph of $f(x) = \cos(x - 60°)$.

There is a phase shift of $60°$ to the right. Since $f(x) = \cos x$ starts has a maximum value when $x = 0°$, the transformed function has a maximum at $x = 60°$ instead.



Questions?