MCR3U: Functions

# Solving Trigonometric Equations

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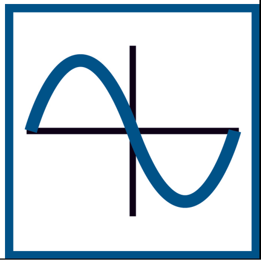





## Solving Trigonometric Equations

Recall that a trigonometric equation, such as $\cos x = \frac{1}{4}$ can be solved for $x$ using the inverse trigonometric functions.

For the case above, $x = \cos^{-1}\left(\frac{1}{4}\right) \approx 75.5°$.

This value lies in quadrant 1. If we were to find all values of $x$ such that $0° \leq x \leq 360°$, then another solution would occur when $x \approx 360° - 75.5° \approx 284.5°$.

It is important to consider both the domain of the variable, as well as the sign of the function (CAST rule) for each given scenario.





## Solving Trigonometric Equations

**Example**

Solve $5\tan x - 2 = 0$, where $0° \leq x \leq 360°$.

Isolate $\tan x$.

$$\begin{aligned} 5\tan x &= 2 \\ \tan x &= \tfrac{2}{5} \end{aligned}$$

Since tangent is positive in quadrants 1 and 3, $x \approx \tan^{-1}\left(\frac{2}{5}\right) \approx 21.8°$ or $x \approx 180° + 201.8°$.





## Solving Trigonometric Equations

**Example**

Solve $3\sin x + 1 = 0$, where $0° \leq x \leq 180°$.

Isolate $\sin x$.

$$\begin{aligned} 3\sin x &= -1 \\ \sin x &= -\tfrac{1}{3} \end{aligned}$$

Sine is negative in quadrants 3 and 4, but the domain is restricted to quadrants 1 and 2.

Therefore, there are no solutions to the equation.





## Solving Trigonometric Equations

**Example**

Solve $5\tan x - 1 = 8\tan x + 4$, where $0° \leq x \leq 180°$.

Collect like terms, then solve.

$$\begin{aligned} 8\tan x - 5\tan x &= -1 - 4 \\ 3\tan x &= -5 \\ \tan x &= -\tfrac{5}{3} \end{aligned}$$

While tangent is negative in quadrants 2 and 4, the domain is restricted to quadrants 1 and 2.

Thus, the only solution is in quadrant 2, when $x = \tan^{-1}\left(-\frac{5}{3}\right) \approx 121°$.





## Solving Trigonometric Equations

**Example**

Solve $2\cos^2 x + \cos x - 1 = 0$, where $0° \leq x \leq 360°$.

Factor like a quadratic.

$$\begin{aligned} 2\cos^2 x + 2\cos x - \cos x - 1 &= 0 \\ 2\cos x(\cos x + 1) - 1(\cos x + 1) &= 0 \\ (2\cos x - 1)(\cos x + 1) &= 0 \end{aligned}$$

When $2\cos x - 1 = 0$, $x = \cos^{-1}\left(\frac{1}{2}\right)$, so $x = 60°$ or $x = 300°$.

When $\cos x + 1 = 0$, $x = \cos^{-1}(-1)$, so $x = 180°$.

In this case, there are three solutions.

## Solving Trigonometric Equations

**Example**

Solve $5\sin^2 x - 4\sin x - 2 = 0$, where $0° \le x \le 360°$.

Since the equation does not factor, use the quadratic formula.

$$\begin{aligned}\sin x &= \frac{4 \pm \sqrt{(-4)^2 - 4(5)(-2)}}{2(5)}\\ &= \frac{4 \pm \sqrt{56}}{10}\\ &= \frac{2 \pm \sqrt{14}}{5}\\ &\approx 1.15, -0.35\end{aligned}$$





## Solving Trigonometric Equations

Since 1.15 is outside of the range for sine, there are no solutions to $\sin x = \frac{2+\sqrt{14}}{5}$.

When $\sin x = \frac{2-\sqrt{14}}{5}$, $x \approx \sin^{-1}(-0.35) \approx 339.6°$, or $x \approx 200.4°$.

In this case, there are only two solutions.





## Questions?