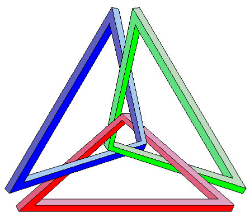
MCR3U: Functions

# Sine and Cosine Laws

J. Garvin

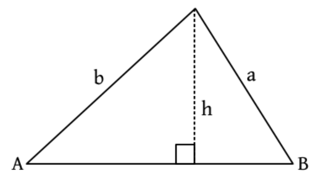






## Sine Law

Consider the diagram shown, where $\angle A$, $\angle B$ and $b$ are known.




How can we determine the length of $a$?





## Sine Law

In the leftmost triangle, $\sin A = \frac{h}{b}$, so $h = b \sin A$.

In the rightmost triangle, $\sin B = \frac{h}{a}$, so $h = a \sin B$.

Using substitution, $b \sin A = a \sin B$, or $\frac{\sin A}{a} = \frac{\sin B}{b}$.

**Law of Sines**

Given $\Delta ABC$, $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$.

The Sine Law is used when information is given about angles and their opposite sides.

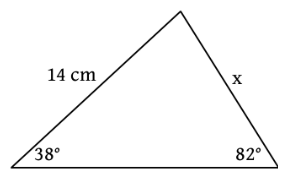




## Sine Law

**Example**

Determine the length of $x$ in the diagram below.








## Sine Law

Use the Sine Law to solve for $x$.

$$\frac{\sin 38^\circ}{x} = \frac{\sin 82^\circ}{14}$$
$$x = \frac{14 \sin 38^\circ}{\sin 82^\circ}$$
$$\approx 8.7 \text{ cm}$$

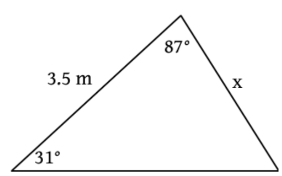




## Sine Law

**Example**

Determine the length of $x$ in the diagram below.



## Sine Law

Since the Sine Law requires angles and their opposite sides, find the measure of the other angle first.

$$180^\circ - 87^\circ - 31^\circ = 62^\circ.$$

$$\begin{aligned} \frac{\sin 31^\circ}{x} &= \frac{\sin 62^\circ}{3.5} \\ x &= \frac{3.5 \sin 31^\circ}{\sin 62^\circ} \\ &\approx 2.04 \text{ m} \end{aligned}$$

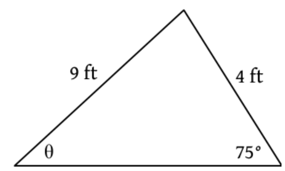




## Sine Law

**Example**

Determine the measure of $\theta$ in the diagram below.








## Sine Law

Use the Sine Law to solve for $\theta$.

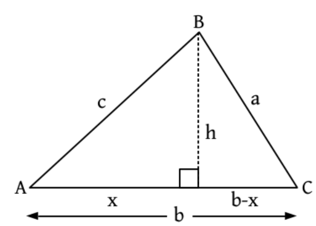
$$\begin{aligned} \frac{\sin \theta}{4} &= \frac{\sin 75^\circ}{9} \\ \sin \theta &= \frac{4 \sin 75^\circ}{9} \\ \theta &= \sin^{-1}\left(\frac{4 \sin 75^\circ}{9}\right) \\ &\approx 25.4^\circ \end{aligned}$$





## Cosine Law

Consider the diagram shown, where $b$, $c$ and $\angle A$ are known.




How can we determine the length of $a$?





## Cosine Law

Using the Pythagorean Theorem In the leftmost triangle, $h^2 = c^2 - x^2$.

Using it in the rightmost triangle, $h^2 = a^2 - (b - x)^2$.

Use substitution and isolate $a^2$.

$$\begin{aligned} a^2 - (b - x)^2 &= c^2 - x^2 \\ a^2 &= c^2 + (b - x)^2 - x^2 \\ &= c^2 + b^2 - 2bx + x^2 - x^2 \\ &= c^2 + b^2 - 2bx \end{aligned}$$





## Cosine Law

In the leftmost triangle, $\cos A = \frac{x}{c}$, so $x = c \cos A$.

Substitute this for $x$ in the above equation.

$$\begin{aligned} a^2 &= c^2 + b^2 - 2bx \\ &= c^2 + b^2 - 2bc \cos A \end{aligned}$$

**Law of Cosines**

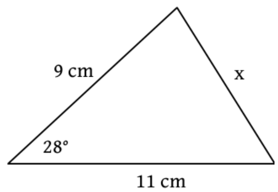
Given $\Delta ABC$, $a^2 = b^2 + c^2 - 2bc \cos A$.

The Cosine Law is used when information is given about an angle and its two adjacent sides, or all three sides.

## Cosine Law

**Example**

Determine the length of $x$ in the diagram below.








## Cosine Law

Use the Cosine Law to solve for $x$.

$$\begin{aligned} x^2 &= 11^2 + 9^2 - 2(11)(9)\cos 28^\circ \\ &= 121 + 81 - 198\cos 28^\circ \\ &= 202 - 198\cos 28^\circ \\ &\approx 202 - 174.8236234 \text{ (BEDMAS)} \\ &\approx 27.17637661 \\ x &\approx \sqrt{27.17637661} \\ &\approx 5.2 \text{ cm} \end{aligned}$$

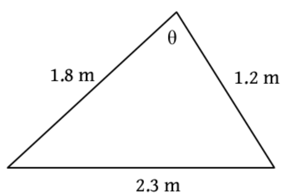




## Cosine Law

**Example**

Determine the measure of $\theta$ in the diagram below.








## Cosine Law

Use the Cosine Law to solve for $\theta$.

$$\begin{aligned} 2.3^2 &= 1.8^2 + 1.2^2 - 2(1.8)(1.2)\cos\theta \\ 5.29 &= 3.24 + 1.44 - 4.32\cos\theta \\ 0.61 &= -4.32\cos\theta \\ \cos\theta &= -\tfrac{0.61}{4.32} \\ &= 0.1412\overline{037} \\ \theta &= \cos^{-1}\left(0.1412\overline{037}\right) \\ &\approx 98^\circ \end{aligned}$$





## Questions?