MCR3U: Functions

Simplifying Rational Expressions

J. Garvin

Rational Expressions

The ratio of two polynomial expressions is called a *rational expression*.

Rational Expressions

Given polynomial expressions $p(x)$ and $q(x)$, a rational expression has the form $\frac{p(x)}{q(x)}$, where $q(x) \neq 0$.

If $q(x) = 0$, then the expression is undefined.

Rational Expressions

A rational expression can be simplified if there are common factors in the numerator and the denominator.

When two factors cancel out, we obtain an *equivalent* expression.

However, it is possible that during the process of simplification, we remove one or more *restrictions* on the variable.

A restriction occurs when a factor in the denominator of the rational expression evaluates to zero.

When simplifying rational expressions, it is important to maintain all restrictions in the variable so that the context of the original expression is preserved.

Simplifying Rational Expressions

Example

Simplify $\frac{(x-3)(x+2)}{(x+1)(x+2)}$.

$$\frac{(x-3)(x+2)}{(x+1)(x+2)} = \frac{(x-3)(x+2)}{(x+1)(x+2)}$$
$$= \frac{x-3}{x+1}, x \neq -2, x \neq -1$$

Remember: the restrictions on x are important!

Simplifying Rational Expressions

Example

Simplify $\frac{5x^2}{20x^3 - 15x^2}$.

The first step is to common factor.

$$\frac{5x^2}{20x^3 - 15x^2} = \frac{5x^2}{5x^2(4x-3)}$$
$$= \frac{1}{4x-3}, x \neq 0, x \neq \frac{3}{4}$$

Again, we must remember to state the restrictions on x or the context of the original expression is lost.

Simplifying Rational Expressions

Example

Simplify $\frac{x^2 - 3x - 10}{x^2 - 2x - 8}$.

Factor the simple trinomials first.

$$\frac{x^2 - 3x - 10}{x^2 - 2x - 8} = \frac{(x-5)(x+2)}{(x-4)(x+2)}$$
$$= \frac{x-5}{x-4}, x \neq -2, x \neq 4$$

Simplifying Rational Expressions

Example

Simplify $\dfrac{3-x}{x^2-2x-3}$.

The first step is to factor the simple trinomial.

$$\begin{aligned}\frac{3-x}{x^2-2x-3} &= \frac{3-x}{(x-3)(x+1)}\\ &= \frac{-1(x-3)}{(x-3)(x+1)}\\ &= -\frac{1}{x+1}, x \neq 3, x \neq -1\end{aligned}$$

It is always possible to factor out -1 by inverting all signs.

Simplifying Rational Expressions

Example

Simplify $\dfrac{2x^2-9x-5}{3x^2-19x+20}$.

Factor the complex trinomials using decomposition.

$$\begin{aligned}\frac{2x^2-9x-5}{3x^2-19x+20} &= \frac{(2x+1)(x-5)}{(3x-4)(x-5)}\\ &= \frac{2x+1}{3x-4}, x \neq 5, x \neq \frac{4}{3}\end{aligned}$$

Simplifying Rational Expressions

Example

Simplify $\dfrac{x^2-5x}{x^2+25}$.

The numerator common factors as $x(x-5)$.

The denominator, on the other hand, is a sum of squares and does not factor. Thus, there are no factors to cancel.

Since $x^2+25 \geq 25$ for all $x \in R$, there are no restrictions on the variable.

Therefore, the rational expression is already simplified.

Simplifying Rational Expressions

Example

Simplify $\dfrac{18-2x^2}{x^2+x-6}$.

Factor everything.

$$\begin{aligned}\frac{18-2x^2}{x^2+x-6} &= \frac{-2(x^2-9)}{(x-2)(x+3)}\\ &= \frac{-2(x-3)(x+3)}{(x-2)(x+3)}\\ &= -\frac{2(x-3)}{(x-2)}, x \neq -3, x \neq 2\end{aligned}$$

Questions?