MCR3U: Functions

Transformations of Functions

Part 2: Reflections, Stretches, Compressions

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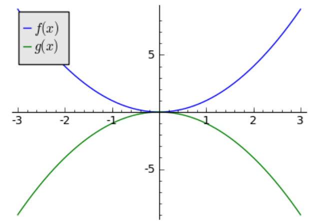
Vertical Reflections

Investigate (Part 1)

On the same grid, graph the functions $f(x) = x^2$ and $g(x) = -x^2$.



Vertical Reflections

A *vertical reflection* is a mirroring of a graph in the x-axis.

Vertical reflections have the form $g(x) = -f(x)$.

A point on the graph, $P(x, y)$, has an image at $P'(x, -y)$ after being vertically reflected.

Vertically reflecting a graph is equivalent to multiplying the graph's equation by -1.

Since multiplication precedes addition and subtraction, vertical reflections must be applied before any vertical translations.

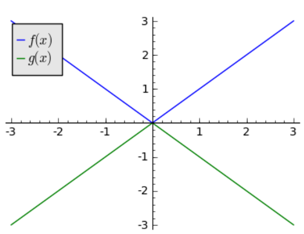
Vertical Reflections

Example

Graph the functions $f(x) = |x|$ and $g(x) = -|x|$, and state the domain and range of $g(x)$.

x	Point	Image
-2	$(-2, 2)$	$(-2, -2)$
-1	$(-1, 1)$	$(-1, -1)$
0	$(0, 0)$	$(0, 0)$
1	$(1, 1)$	$(1, -1)$
2	$(2, 2)$	$(2, -2)$

Vertical Reflections



Domain: $\{x \in R\}$

Range: $\{g(x) \in R \mid g(x) \leq 0\}$

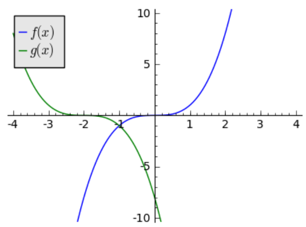
Vertical Reflections

Example

Graph the functions $f(x) = x^3$ and $g(x) = -(x+2)^3$, and state the domain and range of $g(x)$.

x	Point	VR	HT
-2	$(-2, -8)$	$(-2, 8)$	$(-4, 8)$
-1	$(-1, -1)$	$(-1, 1)$	$(-3, 1)$
0	$(0, 0)$	$(0, 0)$	$(-2, 0)$
1	$(1, 1)$	$(1, -1)$	$(-1, -1)$
2	$(2, 8)$	$(2, -8)$	$(0, -8)$

Vertical Reflections



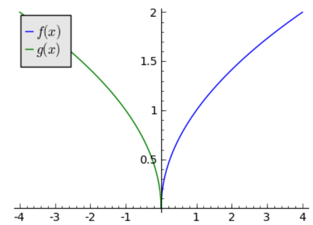
Domain: $\{x \in R\}$

Range: $\{g(x) \in R\}$

Horizontal Reflections

Investigate (Part 2)

On the same grid, graph the functions $f(x) = \sqrt{x}$ and $g(x) = \sqrt{-x}$.



Horizontal Reflections

A *horizontal reflection* is a mirroring of a graph in the y-axis.

Horizontal reflections have the form $g(x) = f(-x)$.

A point on the graph, $P(x, y)$, has an image at $P'(-x, y)$ after being horizontally reflected.

Horizontally reflecting a graph is equivalent to replacing all instances of x in the function's equation with $-x$.

As with vertical reflections, horizontal reflections must be applied before any horizontal translations.

Horizontal Reflections

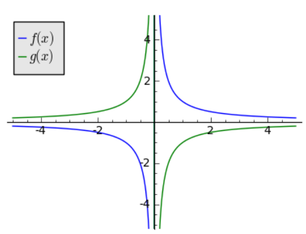
Example

Graph the functions $f(x) = \frac{1}{x}$ and $g(x) = \frac{1}{-x}$, and state the domain and range of $g(x)$.

The horizontal and vertical asymptotes are left unchanged at $y = 0$ and $x = 0$ respectively.

x	Point	Image
-1	$(-1, -1)$	$(1, -1)$
1	$(1, 1)$	$(-1, 1)$

Horizontal Reflections



Domain: $\{x \in R \mid x \neq 0\}$

Range: $\{g(x) \in R \mid g(x) \neq 0\}$

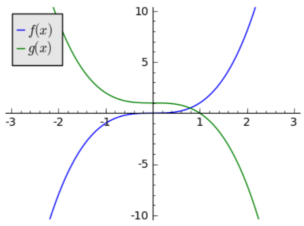
Horizontal Reflections

Example

Graph the functions $f(x) = x^3$ and $g(x) = (-x)^3 + 1$, and state the domain and range of $g(x)$.

x	Point	HR	VT
-2	$(-2, -8)$	$(2, -8)$	$(2, -7)$
-1	$(-1, -1)$	$(1, -1)$	$(1, 0)$
0	$(0, 0)$	$(0, 0)$	$(0, 1)$
1	$(1, 1)$	$(-1, 1)$	$(-1, 2)$
2	$(2, 8)$	$(-2, 8)$	$(-2, 9)$

Horizontal Reflections



Domain: $\{x \in R\}$

Range: $\{g(x) \in R\}$

Vertical and Horizontal Reflections

Example

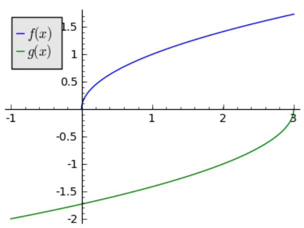
For the function $g(x) = -\sqrt{3-x}$:

- State the parent function.
- Describe the transformations.
- Sketch a graph.
- Determine the domain and range.

The parent function is $f(x) = \sqrt{x}$.

Another way to write $g(x)$ is $g(x) = -\sqrt{-(x-3)}$. This makes it clear that there is a horizontal translation 3 units to the right, in addition to the vertical and horizontal reflections.

Vertical and Horizontal Reflections



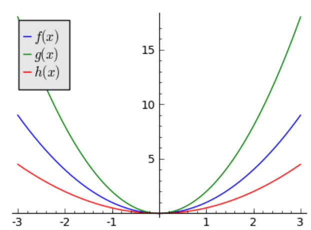
Domain: $\{x \in R \mid x \leq 3\}$

Range: $\{g(x) \in R \mid g(x) \leq 0\}$

Vertical Stretches and Compressions

Investigate (Part 3)

On the same grid, graph the functions $f(x) = x^2$, $g(x) = 2x^2$ and $h(x) = \frac{1}{2}x^2$.



Vertical Stretches and Compressions

Stretches and *compressions* are transformation change the shape of a graph, but not its position.

Vertical stretches and compressions have the form $g(x) = af(x)$.

- When $0 < |a| < 1$, the graph is vertically compressed by a factor of a.
- When $|a| > 1$, the graph is vertically stretched by a factor of a.

The absolute value notation is necessary, since the value of a also indicates whether a vertical reflection is present.

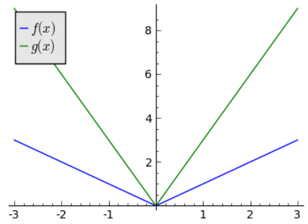
Vertical Stretches and Compressions

Example

Graph the functions $f(x) = |x|$ and $g(x) = 3|x|$, and state the domain and range of $g(x)$.

x	Point	Image
-2	$(-2, 2)$	$(-2, 6)$
-1	$(-1, 1)$	$(-1, 3)$
0	$(0, 0)$	$(0, 0)$
1	$(1, 1)$	$(1, 3)$
2	$(2, 2)$	$(2, 6)$

Vertical Stretches and Compressions



Domain: $\{x \in R\}$

Range: $\{g(x) \in R \mid g(x) \geq 0\}$

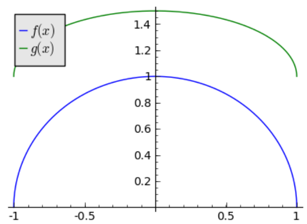
Vertical Stretches and Compressions

Example

Graph the functions $f(x) = \sqrt{1 - x^2}$ and $g(x) = \frac{1}{2}\sqrt{1 - x^2} + 1$, and state the domain and range of $g(x)$.

x	Point	VC	VT
-1	$(-1, 0)$	$(-1, 0)$	$(-1, 1)$
0	$(0, 1)$	$(0, \frac{1}{2})$	$(0, \frac{3}{2})$
1	$(1, 0)$	$(1, 0)$	$(1, 1)$

Vertical Stretches and Compressions



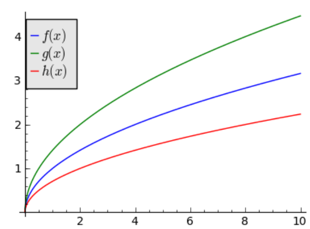
Domain: $\{x \in R \mid -1 \leq x \leq 1\}$

Range: $\left\{g(x) \in R \mid 1 \leq g(x) \leq \frac{3}{2}\right\}$

Horizontal Stretches and Compressions

Investigate (Part 4)

On the same grid, graph the functions $f(x) = \sqrt{x}$, $g(x) = \sqrt{2x}$ and $h(x) = \sqrt{\frac{1}{2}x}$.



Horizontal Stretches and Compressions

Horizontal stretches and compressions have the form $g(x) = f(bx)$.

- When $0 < |b| < 1$, the graph is horizontally stretched by a factor of $\frac{1}{b}$.
- When $|b| > 1$, the graph is horizontally compressed by a factor of $\frac{1}{b}$.

For example, if $b = \frac{1}{3}$, the graph is horizontally stretched by a factor of $\frac{1}{\frac{1}{3}} = 3$. The graph is three times as wide.

If $b = 5$, the graph is horizontally compressed by a factor of $\frac{1}{5}$. The graph is one fifth as wide.

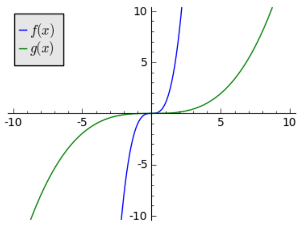
Horizontal Stretches and Compressions

Example

Graph the functions $f(x) = x^3$ and $g(x) = \left(\frac{1}{4}x\right)^3$, and state the domain and range of $g(x)$.

x	Point	Image
-2	$(-2, -8)$	$(-8, -8)$
-1	$(-1, -1)$	$(-4, -1)$
0	$(0, 0)$	$(0, 0)$
1	$(1, 1)$	$(4, 1)$
2	$(2, 8)$	$(8, 8)$

Horizontal Stretches and Compressions



Domain: $\{x \in R\}$
Range: $\{g(x) \in R\}$

Questions?