MCR3U: Functions

Working with Radicals

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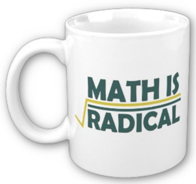

Radicals

A *radical*, also called a *root*, is typically represented using the form $\sqrt[n]{x}$.

The argument x is called the *radicand*, while n is called the *index*.

For example, the third root (or *cube root*) of 5 is written $\sqrt[3]{5}$, and means "the value which, when multiplied by itself three times, gives five."

If no index is specified, the square root ($\sqrt[2]{x}$) is implied.

Mixed Radicals

A *mixed radical* is the product of two components, one involving a radical and one without.

For example, $2\sqrt{6}$ is the same as writing $2 \times \sqrt{6}$ or $(2)(\sqrt{6})$.

We are often interested in "simplifying" radicals by writing them as mixed radicals, thus reducing the value of the radicand.

Mixed Radicals

Example

Express $\sqrt{40}$ as a mixed radical.

Expressing 40 as the product of two integers, where at least one is a square number, we get $40 = 4 \times 10$.

$$\begin{aligned}\sqrt{40} &= \sqrt{4 \times 10}\\ &= \sqrt{4}\sqrt{10}\\ &= 2\sqrt{10}\end{aligned}$$

Mixed Radicals

Your Turn

Express $\sqrt{63}$ as a mixed radical.

$$\begin{aligned}\sqrt{63} &= \sqrt{9 \times 7}\\ &= \sqrt{9}\sqrt{7}\\ &= 3\sqrt{7}\end{aligned}$$

Mixed Radicals

Example

Express $\sqrt{128}$ as a mixed radical.

If the greatest square number is not obvious, try reducing using multiple steps.

$$\begin{aligned}\sqrt{128} &= \sqrt{4 \times 32}\\ &= \sqrt{4}\sqrt{32}\\ &= 2\sqrt{32}\\ &= 2\sqrt{16 \times 2}\\ &= 2\sqrt{16}\sqrt{2}\\ &= (2 \times 4)\sqrt{2}\\ &= 8\sqrt{2}\end{aligned}$$

Mixed Radicals

Your Turn

Express $\sqrt{252}$ as a mixed radical.

$$\begin{aligned}\sqrt{252} &= \sqrt{4 \times 63}\\ &= \sqrt{4}\sqrt{63}\\ &= 2\sqrt{63}\\ &= 2\sqrt{9 \times 7}\\ &= 2\sqrt{9}\sqrt{7}\\ &= (2 \times 3)\sqrt{7}\\ &= 6\sqrt{7}\end{aligned}$$

Simplifying Expressions Involving Radicals

Recall that like terms in polynomial expressions have the same variables with the same exponents.

Similarly, radicals that have the same radicand can be treated as like terms, and can be added or subtracted as necessary.

For example, we know that $3x + 4x = 7x$. In the same manner, $3\sqrt{2} + 4\sqrt{2} = 7\sqrt{2}$.

Radicals with unlike radicands cannot be combined, unless they can be converted to like radicands.

Simplifying Expressions Involving Radicals

Example

Simplify $2\sqrt{5} + 7\sqrt{5}$.

Each radical has a radicand of 5, so the two terms can be combined.

$$\begin{aligned}2\sqrt{5} + 7\sqrt{5} &= (2 + 7)\sqrt{5}\\ &= 9\sqrt{5}\end{aligned}$$

Simplifying Expressions Involving Radicals

Example

Simplify $3\sqrt{20} - 9\sqrt{5}$.

Begin by finding a common radicand.

$$\begin{aligned}3\sqrt{20} - 9\sqrt{5} &= 3\sqrt{4 \times 5} - 9\sqrt{5}\\ &= (3 \times 2)\sqrt{5} - 9\sqrt{5}\\ &= 6\sqrt{5} - 9\sqrt{5}\\ &= -3\sqrt{5}\end{aligned}$$

Simplifying Expressions Involving Radicals

Your Turn

Simplify $3\sqrt{32} + 10\sqrt{8}$.

$$\begin{aligned}3\sqrt{32} + 10\sqrt{8} &= 3\sqrt{16 \times 2} + 10\sqrt{4 \times 2}\\ &= (3 \times 4)\sqrt{2} + (10 \times 2)\sqrt{2}\\ &= 12\sqrt{2} + 20\sqrt{2}\\ &= 32\sqrt{2}\end{aligned}$$

Simplifying Expressions Involving Radicals

Example

Expand and simplify $(3 + \sqrt{5})(2 - \sqrt{5})$.

Use the distributive property, as with any two binomials.

$$\begin{aligned}(3 + \sqrt{5})(2 - \sqrt{5}) &= 3 \times 2 - 3\sqrt{5} + 2\sqrt{5} - \sqrt{5}\sqrt{5}\\ &= 6 - \sqrt{5} - 5\\ &= 1 - \sqrt{5}\end{aligned}$$

Simplifying Expressions Involving Radicals

Your Turn

Expand and simplify $(5+2\sqrt{18})(3+7\sqrt{8})$.

Both $\sqrt{18}$ and $\sqrt{8}$ can be simplified.

$$\begin{aligned}(5+2\sqrt{18})(3+7\sqrt{8}) =& (5+6\sqrt{2})(3+14\sqrt{2})\\ =& 5\times 3+(5\times 14)\sqrt{2}+(6\times 3)\sqrt{2}+\\ & (6\times 14)\sqrt{2}\sqrt{2}\\ =& 15+70\sqrt{2}+18\sqrt{2}+168\\ =& 183+88\sqrt{2}\end{aligned}$$

Rationalizing Radical Denominators

Sometimes, we encounter rational expressions that have radicals in their denominators.

A mathematical convention is to use equivalent expressions that eliminate the radicals from the denominators.

The process of converting a rational expression to one without radicals in its denominator is called *rationalizing the denominator*.

Rationalizing Radical Denominators

Example

Rationalize $\dfrac{3\sqrt{2}}{\sqrt{5}}$.

Multiply both the numerator and denominator by $\sqrt{5}$.

$$\begin{aligned}\frac{3\sqrt{2}}{\sqrt{5}} &= \frac{3\sqrt{2}\sqrt{5}}{\sqrt{5}\sqrt{5}}\\ &= \frac{3\sqrt{10}}{5}\end{aligned}$$

Note that we cannot reduce the 10 and the 5, since 10 is the radicand and not a factor.

Rationalizing Radical Denominators

Your Turn

Rationalize $\dfrac{6\sqrt{5}}{\sqrt{3}}$.

Multiply both the numerator and denominator by $\sqrt{3}$.

$$\begin{aligned}\frac{6\sqrt{5}}{\sqrt{3}} &= \frac{6\sqrt{5}\sqrt{3}}{\sqrt{3}\sqrt{3}}\\ &= \frac{6\sqrt{15}}{3}\\ &= 2\sqrt{15}\end{aligned}$$

In this case, the denominator disappears completely!

Rationalizing Radical Denominators

Example

Rationalize $\dfrac{4-5\sqrt{3}}{1+\sqrt{2}}$.

Multiply both the numerator and denominator by the *conjugate*, $1-\sqrt{2}$.

$$\begin{aligned}\frac{4-5\sqrt{3}}{1+\sqrt{2}} &= \frac{4-5\sqrt{3}}{1+\sqrt{2}}\times\frac{1-\sqrt{2}}{1-\sqrt{2}}\\ &= \frac{4\times 1-4\sqrt{2}-5\sqrt{3}+5\sqrt{3}\sqrt{2}}{1-\sqrt{2}+\sqrt{2}-\sqrt{2}\sqrt{2}}\\ &= \frac{4-4\sqrt{2}-5\sqrt{3}+5\sqrt{6}}{1-2}\\ &= -4+4\sqrt{2}+5\sqrt{3}-5\sqrt{6}\end{aligned}$$

Questions?