MCR3U: Functions

Periodic Functions

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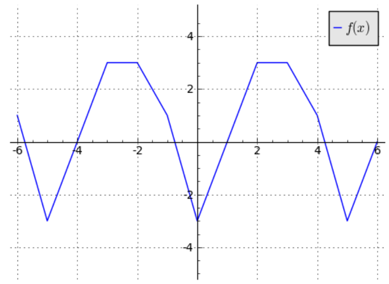
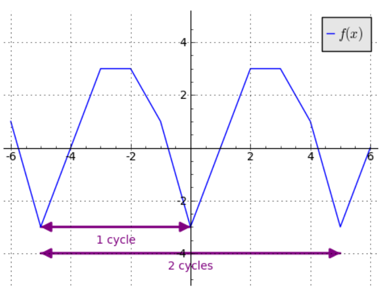
Periodic Behaviour

Consider the graph below. What are some properties of the graph?



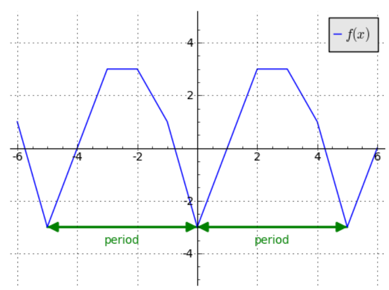
Periodic Behaviour

The graph repeats at regular intervals. A function with this property is called a *periodic function*. One such interval is called a *cycle*.



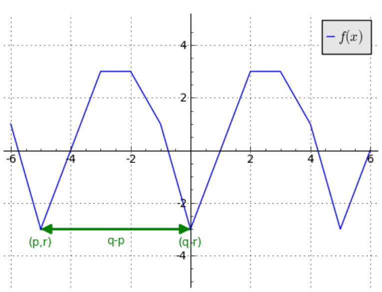
Periodic Behaviour

The length of one cycle is called the *period*. In this case, the period is 5 units.



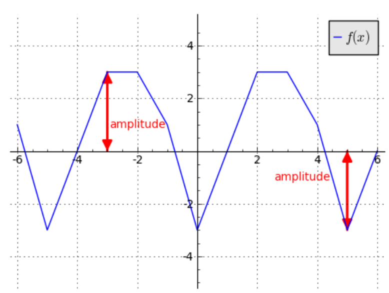
Periodic Behaviour

A periodic function that completes one cycle between points (p, r) and (q, r) has a period of $q - p$.



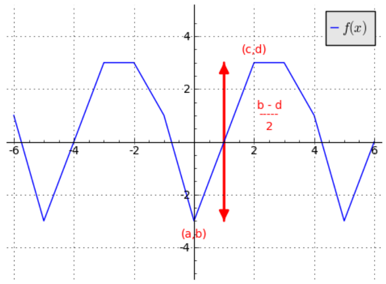
Periodic Behaviour

The minimum value is -3 and the maximum is 3. Half of the distance between these values is called the *amplitude*. The amplitude of this function is 3 units.



Periodic Behaviour

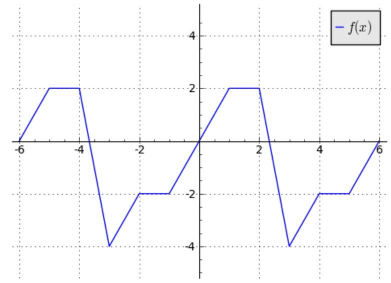
A periodic function with minimum value (a, b) and maximum value (c, d) has an amplitude of $\left|\frac{b-d}{2}\right|$.



Periodic Behaviour

Example

For the function below, state the period and amplitude.



Periodic Behaviour

The function has a minimum value at $(-3, -4)$ and again at $(3, -4)$.

Since the function repeats after every minimum value, the period is $3 - (-3) = 6$ units.

Alternatively, the maximum values could be used instead.

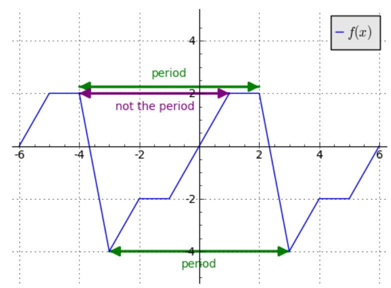
There is a maximum value at $(-4, 2)$ and another at $(1, 2)$.

The point at $(1, 2)$, however, is not the point after which the function repeats. There is a small horizontal section that occurs first.

Instead, use the point $(2, 2)$. This yields the same period of $2 - (-4) = 6$ units.

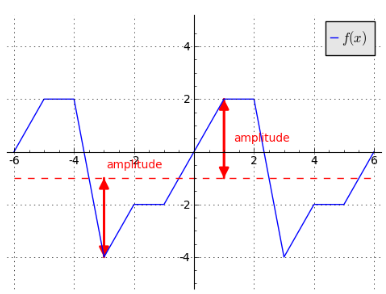
Periodic Behaviour

The diagram below shows the period using both the minimum and maximum values.



Periodic Behaviour

Since the minimum value of the function is -4 and the maximum is 2, the amplitude is $\left|\frac{-4-2}{2}\right| = 3$.



Predictions Using Periodic Behaviour

Since a periodic function repeats regularly, it is possible to predict values that may occur earlier or later.

For example, if a function had a period of 5 units, then $f(0) = f(5) = f(10) = \ldots = f(5n)$.

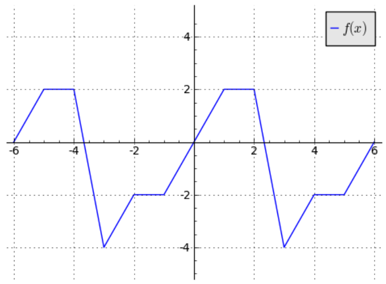
Similarly, $f(1) = f(-4) = f(-9) = \ldots = f(1 - 5n)$.

In each of these cases, the value of the period is repeatedly added or subtracted from a known value of the function.

Predictions Using Periodic Behaviour

Example

For the function below, determine $f(14)$, $f(-19)$ and $f(600)$.



Predictions Using Periodic Behaviour

The period is 6, so we must add or subtract 6 until we reach a known value on the graph.

$f(14) = f(6+6+2) = f(2) = 2$.

$f(-19) = f(-1-6-6-6) = f(-1) = -2$.

$f(600) = f(6 \times 100) = f(0 + \underbrace{6+6+\ldots+6}_{100 \text{ times}}) = f(0) = 0$.

Questions?