MCR3U: Functions

Transformations of Functions

Part 3: Combinations of Multiple Transformations

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Transformations

As we have seen, transformations may act vertically or horizontally.

Vertical transformations are represented by parameters on the *outside* of a function's equation, whereas horizontal transformations have their parameters *inside* of its equation.

The three transformations we have discussed are:

- Translations (movement)
- Reflections (orientation)
- Stretches/Compressions (shape)

Transformations

A parent function, $f(x)$, may have multiple transformations applied to it.

For example, the function $f(x) = -\sqrt{5x} + 1$ has been transformed in three ways:

- Reflected vertically (in the x-axis).
- Compressed horizontally by a factor of $\frac{1}{5}$.
- Translated vertically upward by 1 unit.

Transformations

We can generalize for any parent function that has undergone the transformations mentioned earlier.

General Equation of a Transformed Function

A parent function, $f(x)$, that has been transformed into a new function, $g(x)$ has the form $g(x) = af(b(x-c)) + d$, where:

- f is the parent function
- a is a vertical stretch or compression, or a reflection, or both;
- b is a horizontal stretch or compression, or a reflection, or both;
- c is a horizontal translation, and
- d is a vertical translation.

Transformations

Example

The function $f(x) = x^3$ has three transformations applied to it to produce a new function $g(x)$:

- Vertical stretch by a factor of 5.
- Reflection in the $f(x)$-axis.
- Horizontal translation 2 units left.

Determine an equation for $g(x)$.

Using the general equation, $a = 5$, $b = -1$ and $c = -2$.

Therefore, $g(x) = 5(-(x+2))^3$. Note the apparent sign change for c.

$g(x) = 5(-x-2)^3$ or $g(x) = -5(x+2)^3$ as well, but this expansion is not necessary.

Transformations

Example

The function $f(x) = 2^x$ has four transformations applied to it to produce a new function $g(x)$:

- Vertical compression by a factor of $\frac{2}{3}$.
- Vertical reflection.
- Horizontal translation 5 units right.
- Vertical translation 7 units down.

Determine an equation for $g(x)$.

$a = -\frac{2}{3}$, $c = 5$ and $d = -7$.

Therefore, $g(x) = -\frac{2}{3} \cdot 2^{x-5} - 7$.

Since a horizontal translation is *inside* of a function, it is inside of the exponent in 2^x.

Transformations

Example

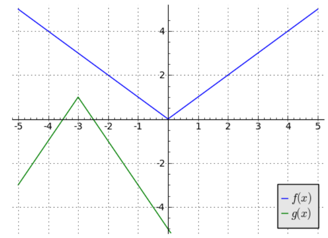
Identify the transformations applied to $f(x) = |x|$ to produce $g(x) = -2|x+3| + 1$, graph $g(x)$, and state the domain and range.

There are four transformations:

- Vertical reflection.
- Vertical stretch by a factor of 2.
- Horizontal translation 3 units left.
- Vertical translation 1 unit up.

Remember to apply all multiplications (stretches, compressions, reflections) before additions (translations).

Transformations



Domain: $\{x \in R\}$

Range: $\{g(x) \in R \mid g(x) \leq 1\}$

Transformations

Example

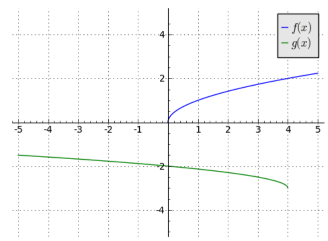
Identify the transformations applied to $f(x) = \sqrt{x}$ to produce $g(x) = \frac{1}{2}\sqrt{-x+4} - 3$, graph $g(x)$, and state the domain and range.

Remember to factor the equation: $g(x) = \frac{1}{2}\sqrt{-(x-4)} - 3$.

There are four transformations:

- Vertical compression by a factor of $\frac{1}{2}$.
- Horizontal reflection.
- Horizontal translation 4 units *right*.
- Vertical translation 3 units down.

Transformations



Domain: $\{x \in R \mid x \leq 4\}$

Range: $\{g(x) \in R \mid g(x) \geq -3\}$

Transformations

Example

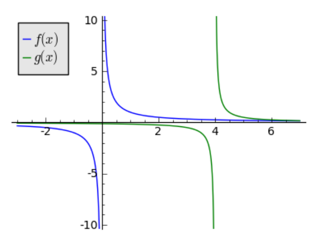
Graph the functions $f(x) = \frac{1}{x}$ and $g(x) = \frac{1}{2x-8}$, and state the domain and range of $g(x)$.

Rewrite the function as $g(x) = \frac{1}{2(x-4)}$.

The horizontal asymptote shifts 4 units right, to $x = 4$. The vertical asymptote remains unchanged at $y = 0$.

x	Point	HC	HT
-1	$(-1,-1)$	$(-\frac{1}{2},-1)$	$(\frac{7}{2},-1)$
1	$(1,1)$	$(\frac{1}{2},1)$	$(\frac{9}{2},1)$

Transformations



Domain: $\{x \in R \mid x \neq 4\}$

Range: $\{g(x) \in R \mid g(x) \neq 0\}$

Transformations

Example

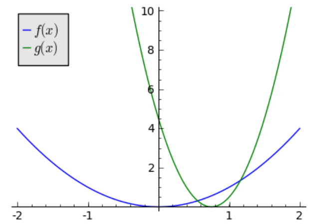
For the function $g(x) = \frac{1}{2}(4x - 3)^2$:

- State the parent function.
- Describe the transformations.
- Sketch a graph.
- Determine the domain and range.

The parent function is $f(x) = x^2$.

Rewrite the equation as $g(x) = \frac{1}{2}\left(4\left(x - \frac{3}{4}\right)\right)^2$. There is a vertical compression by a factor of $\frac{1}{2}$, a horizontal compression by a factor of $\frac{1}{4}$, and a horizontal translation of $\frac{3}{4}$ units to the right.

Transformations



Domain: $\{x \in R\}$

Range: $\{g(x) \in R \mid g(x) \geq 0\}$

Questions?