MCR3U: Functions

# Minimums/Maximums of Quadratic Functions

## Part 2: Word Problems

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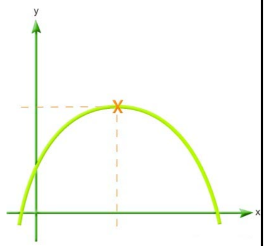







## Min/Max Word Problems

Many problems involve determining the maximum or minimum value of a quadratic function, given some set of constraints.

Typically (but not always), keywords such as "largest", "smallest", "greatest", "least" are indicators to find the maximum/minimum value.

Both completing the square or partial factoring can be used to do this.





## Sum/Product Problems

**Example**

The sum of two numbers is 50. Determine the largest possible product.

Let $x$ and $y$ be the two numbers with a sum of 50.

$$\begin{aligned} x + y &= 50 \\ y &= 50 - x \end{aligned}$$

Let $P(x)$ be the product of the two numbers, expressed in terms of the first number $x$, then find the zeroes.

$$\begin{aligned} P(x) &= x \cdot y \\ 0 &= x(50 - x) \\ x &= 0 \text{ or } 50 \end{aligned}$$





## Sum/Product Problems

The average of 0 and 50 is $\frac{0+50}{2} = 25$.

$$\begin{aligned} P(25) &= 25(50 - 25) \\ &= 25 \cdot 25 \\ &= 625 \end{aligned}$$

Therefore, the maximum possible product is 625, when the two numbers are both 25.





## Sum/Product Problems

The same answer could have been reached by completing the square.

$$\begin{aligned} P(x) &= x(50 - x) \\ &= -x^2 + 50x \\ &= -(x^2 - 50x) \\ &= -(x^2 - 50x + 625 - 625) \\ &= -(x - 25)^2 + 625 \end{aligned}$$

Either method is acceptable, so use the one with which you are most comfortable.



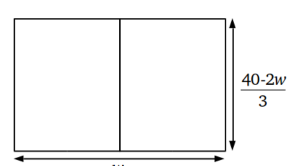


## Area Problems

**Example**

40 metres of fencing is available to build a rectangular pen that is separated into two equally-sized areas by fencing running parallel to one side. Determine the greatest possible area of the pen.

Drawing a diagram helps.



## Area Problems

Let $A(w)$ be the area of a rectangle, expressed in terms of its width.

$$A = l \cdot w$$
$$0 = w\left(\frac{40-2w}{3}\right)$$
$$w = 0 \text{ or } 20$$

The average of 0 and 20 is $\frac{0+20}{2} = 10$.

$$A(10) = 10\left(\frac{40-2(10)}{3}\right)$$
$$= \frac{200}{3} \text{ or } 66.\overline{6}$$

The greatest possible area of $66.\overline{6}$ m$^2$ occurs when the width is 10 m and the length is $\frac{20}{3} = 6.\overline{6}$ m.





## Increase/Decrease Problems

**Example**

Using its website, a band typically sells an average of 100 t-shirts per month for \$20 each. They discover that for every \$1 decrease in price, they sell 10 more shirts each month. Determine the greatest revenue that the band can generate each month, and the selling price of a t-shirt.

Let $n$ be the number of \$1 decreases, and $R(n)$ the revenue.

$$R(n) = (100 + 10n)(20 - n)$$
$$0 = 10(10 + n)(20 - n)$$
$$n = -10 \text{ or } 20$$





## Increase/Decrease Problems

The average of $-10$ and 20 is $\frac{-10+20}{2} = 5$.

$$R(5) = 10(10 + 5)(20 - 5)$$
$$= 2\,250$$

The maximum revenue is \$2 250 when there are five \$1 decreases, for a selling price of \$15.





## Questions?