MCR3U: Functions

Graphs of Sine, Cosine and Tangent Functions

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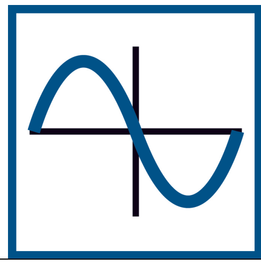


Graphing $f(x) = \sin x$

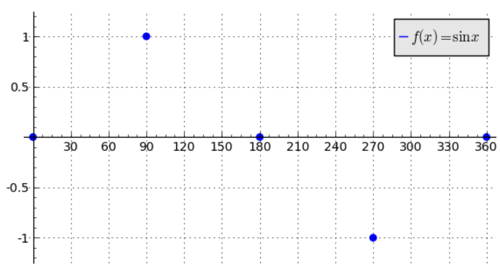
What does the graph of $f(x) = \sin x$ look like?

We know the exact values for $\sin\theta$ when θ is 0°, 90°, 180°, 270° and 360°.

θ	0°	90°	180°	270°	360°
$\sin\theta$	0	1	0	-1	0

Graphing $f(x) = \sin x$

Plotting these five points results in the following graph.



The function appears to rise and fall, but some additional points might make the pattern clearer.

Graphing $f(x) = \sin x$

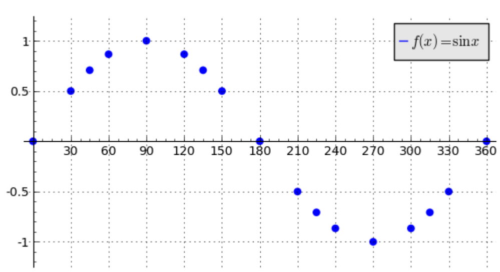
Add to our graph the exact values when θ is 30°, 45°, 60°, and so on into quadrants 2-4.

θ	30°	45°	60°	120°	135°	150°
$\sin\theta$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$

θ	210°	225°	240°	300°	315°	330°
$\sin\theta$	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$

Graphing $f(x) = \sin x$

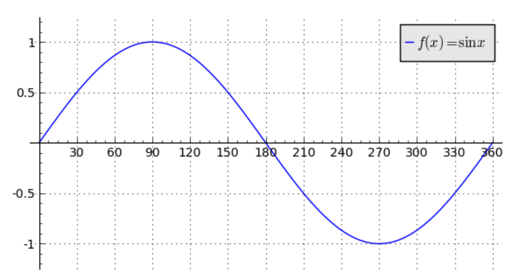
Adding these points results in the following graph.



The function makes a wave-like form, called a *sine wave*.

Graphing $f(x) = \sin x$

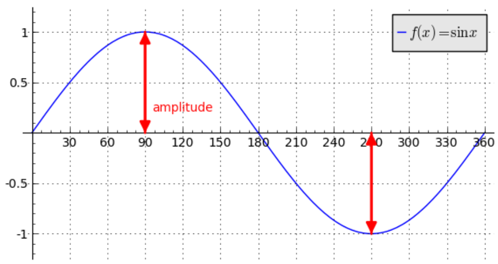
A complete graph of $f(x) = \sin x$ is below.



The function has several important properties.

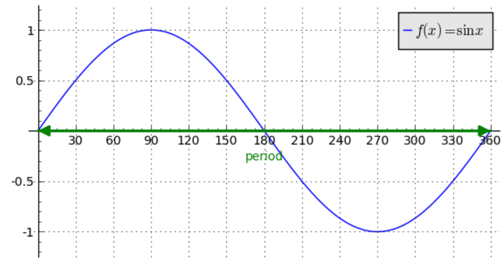
Properties of $f(x) = \sin x$

For a *sinusoidal function* like $f(x) = \sin x$, the *amplitude* of the function is half the difference of the maximum value and the minimum value.



Properties of $f(x) = \sin x$

The *period* of a sinusoidal function is the amount of time it takes to complete one full *cycle*.



Properties of $f(x) = \sin x$

The function $f(x) = \sin x$ has the following key properties:

- The $f(x)$-intercept is at 0.
- There are x-intercepts at $x = 0°, 180°, \ldots, (180n)°$.
- The domain is $\{x \in R\}$.
- The range is $\{f(x) \in R \mid -1 \leq f(x) \leq 1\}$.
- The amplitude is 1.
- The period is 360°.

Graphing $f(x) = \cos x$

We can repeat the process for $f(x) = \cos x$, plotting exact values until a shape emerges.

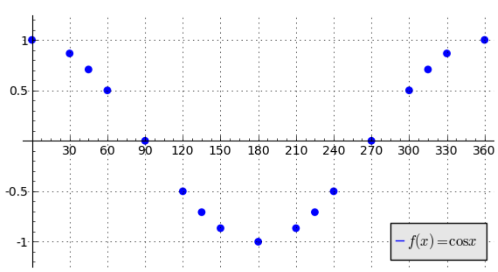
θ	0°	90°	180°	270°	360°
$\cos\theta$	1	0	-1	0	1

θ	30°	45°	60°	120°	135°	150°
$\cos\theta$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$

θ	210°	225°	240°	300°	315°	330°
$\cos\theta$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$

Graphing $f(x) = \cos x$

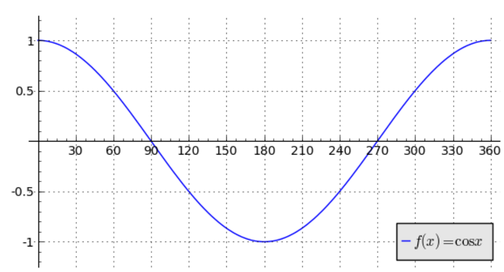
Plotting these points results in the following graph.



The graph of $f(x) = \cos x$ also makes a sine wave.

Graphing $f(x) = \cos x$

A complete graph of $f(x) = \cos x$ is below.



Like sine, the graph of cosine has several important properties.

Properties of $f(x) = \cos x$

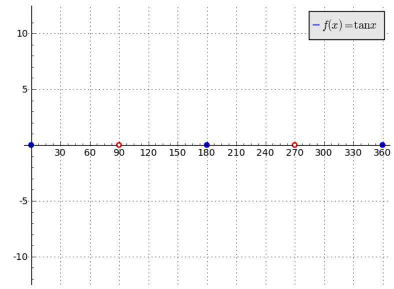
The function $f(x) = \cos x$ has the following key properties:

- The $f(x)$-intercept is at 1.
- There are x-intercepts at $x = 90^\circ, 270^\circ, \ldots, (180n + 90)^\circ$.
- The domain is $\{x \in R\}$.
- The range is $\{f(x) \in R \mid -1 \leq f(x) \leq 1\}$.
- The amplitude is 1.
- The period is 360°.

Graphing $f(x) = \tan x$

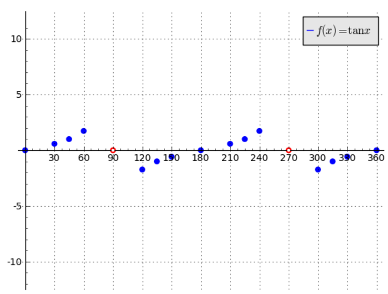
The graph of $f(x) = \tan\theta$ is more problematic.

$\tan\theta$ is undefined when θ is 90° or 270°, and 0 when θ is 0°, 180° or 360°, a graph of these points is severely limited.



Graphing $f(x) = \tan x$

Adding more points when θ is 30°, 45°, 60°, and so on gives some additional insight, but still not enough.



Graphing $f(x) = \tan x$

Choose some values on either side of 90°.

θ	80°	85°	89°
$\tan\theta$	5.67	11.43	57.29

θ	91°	95°	100°
$\tan\theta$	-57.29	-11.43	-5.67

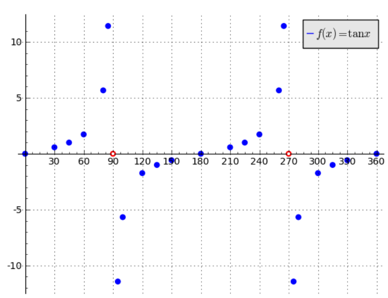
As θ increases toward 90° from the left, $\tan\theta$ becomes very large.

As θ increases toward 90° from the right, $\tan\theta$ becomes very small.

The same thing occurs using values close to 270°.

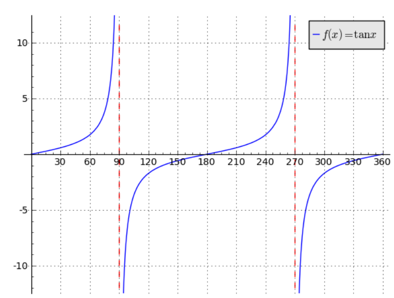
Graphing $f(x) = \tan x$

Putting things together, we get the following graph.



Graphing $f(x) = \tan x$

A complete graph of $f(x) = \tan x$ is below.



Properties of $f(x) = \tan x$

The function $f(x) = \tan x$ has the following key properties:

- The $f(x)$-intercept is at 0.
- There are x-intercepts at $x = 0°, 180°, \ldots, (180n)°$.
- The domain is $\{x \in R \mid x \neq 180n + 90\}$ (there are vertical asymptotes at these points).
- The range is $\{f(x) \in R\}$.
- There is no amplitude (stretches infinitely).
- The period is $180°$.

Questions?